Journal Homepage: www.ijarpr.com ISSN: 3049-0103 (Online)

International Journal of Advance Research Publication and Reviews

Vol 02, Issue 12, pp 60-81, December, 2025

A Review: Roadef/Euro 2016, Inventory Routing Problem

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ABSTRACT

This study offers a detailed examination and critique of the academic methods, tools, and algorithms used in inventory routing and supply chain management. It highlights the major challenges researchers face, ranging from the complexity of real-world operational environments to the intricate interactions among multiple constraints. These challenges underscore the need for scalable, flexible solutions that can meet the growing demands of modern supply chains. The review also draws attention to limitations within existing research approaches. In many studies, difficulties arise from handling constraints and uncertainties, while others rely on overly rigid or highly precise problem formulations that do not reflect real-world variability. By identifying these shortcomings, the paper provides valuable insights for researchers both by revealing the specific obstacles faced in past studies and by offering a broader understanding of how different methods can be applied across diverse contexts.

Ultimately, this analysis serves as a catalyst for further research. By outlining the gaps and weaknesses in current study designs, it encourages the development of new algorithms, the refinement of existing models, and the integration of advanced optimization techniques. It also points to the rising importance of hybrid methodologies that combine strengths from multiple approaches. As researchers continue to pursue efficiency and sustainability in supply chain management, they are increasingly adapting their models and algorithms to incorporate real-world uncertainties, environmental considerations, and practical constraints. Areas such as robust optimization, population-based methods, and collaborative techniques present significant opportunities for innovation in algorithm design and problem-solving strategies. To develop adaptable solution strategies that perform well across varied real-world scenarios remains a central priority. The ongoing evolution of research in inventory routing signals a promising future one in which supply chain systems become more resilient, efficient, and sustainable.

1.0 Introduction

Advances in Artificial Intelligence, especially machine learning, have strengthened efforts to solve the Inventory Routing Problem (IRP), a key challenge in reducing supply chain costs and improving efficiency (Islam & Wasi, 2023). Under Vendor Managed Inventory (VMI), vendors monitor customer consumption, forecast replenishment needs, and coordinate delivery routes, making routing vehicles central to supply chain operations (Munyaka et al., 2022; Agra et al., 2018; Cao et al., 2020).

IRP focuses on planning routes and delivery quantities that maintain optimal inventory levels while meeting customer demand. The complexity of these decisions has motivated diverse algorithmic solutions, including high-performing hyper-heuristic methods (Kheiri et al., 2016). Integrated optimisation of inventory and transportation is now considered essential for reducing overall costs and preventing inefficiencies caused by treating these components independently (Kheiri & Zografos, 2019).

VMI plays a pivotal role by fostering collaboration between suppliers and buyers, reducing the bullwhip effect, and enhancing supply chain performance (Laganà et al., 2016). Understanding IRP dynamics remains crucial for designing efficient, adaptive supply chain systems and guiding future research in optimisation (Kheiri, 2020). The goals of this review are outlined as follows:

- **Identifying Challenges and Weaknesses:** Reviews serve as catalysts for improvement by analysing and evaluating the methods, tools, and algorithms proposed by researchers to address complex inventory routing and supply chain management problems (Bharadiya, 2023). Through careful examination of existing studies, the challenges encountered by researchers become clearer. These include the complexity of real-world operational environments, the interaction of multiple constraints, and the need for scalable, adaptable solutions. The review process also exposes the limitations of various research approaches—such as algorithmic shortcomings, narrow problem formulations, and difficulties in handling uncertainties or specific constraints. Recognizing these gaps is essential for developing stronger methodologies and more effective solutions (Laganà et al., 2016).

- **Informed Decision-Making for Future Research:** As highlighted by Aderibigbe et al. (2023), identifying these challenges and limitations helps guide future research directions. The review offers not only detailed insight into the specific obstacles faced by previous scholars but also a broader understanding of how different methods perform across various contexts. By leveraging the strengths of existing techniques while addressing their weaknesses, researchers can design innovative approaches to overcome past limitations (Islam & Wasi, 2023). This informed decision-making ensures more efficient use of research efforts and reduces the likelihood of repeating previous mistakes (Kheiri, 2020).

- **Enhancing Methodologies and Approaches:** The problems and inadequacies identified in the literature act as a driving force for innovation and methodological enhancement (Aderibigbe et al., 2023). With a clear understanding of past challenges, researchers are better positioned to develop improved solutions (Bharadiya, 2023). Progress in inventory routing and supply chain management can be accelerated by building on earlier studies, refining existing models, and introducing stronger algorithmic strategies. Collaboration among researchers further enables the development of comprehensive frameworks capable of addressing the dynamic and complex nature of real-world supply chains (Kheiri & Zografos, 2019).

2.0 ROADEF/EURO and Inventory Routing Problem

The historical background of the ROADEF/EURO 2016 IRP Challenge can be traced back to the early 1990s, when Air Liquide began applying the Vehicle Routing Problem (VRP) to optimize its delivery operations (André et al., 2020). As a global leader in industrial gas production and distribution, Air Liquide manages a vast fleet responsible for delivering gases to customers worldwide. In its effort to improve delivery efficiency, the company initially adopted VRP techniques, which at the time appeared to be a promising solution.

However, Air Liquide soon discovered that traditional VRP formulations were insufficient for its operational realities. The VRP did not incorporate inventory management considerations an essential aspect of gas distribution, where customer demand and storage levels must be constantly monitored. This limitation led the company to conceptualize the Inventory Routing Problem (IRP), a more comprehensive model that integrates both routing decisions and inventory replenishment planning. Although the IRP has been explored in operations research for several decades (Islam & Wasi, 2023), it remains a highly complex and computationally challenging problem. No universal algorithm guarantees optimal solutions across all IRP variations. To encourage innovation and identify effective solution strategies, the ROADEF/EURO 2016 IRP Challenge was organised as a platform to stimulate research and evaluate cutting-edge optimisation techniques.

The challenge consisted of two stages:

- **Qualification Stage:** Participants were required to solve 10 IRP instances designed to reflect real-world complexity. These instances varied in the number of customers, available vehicles, and inventory requirements, offering a broad representation of practical scenarios (Obaid, 2018).

- **Final Stage:** Teams then tackled a single, more difficult IRP instance intended to test the robustness, adaptability, and efficiency of their solution methods (Obaid, 2018). The final evaluation was based solely on performance on this challenging instance.

The winning team successfully applied a hybrid metaheuristic approach that combined local search with a genetic algorithm. This hybrid method demonstrated strong computational performance and the flexibility required to navigate the multi-layered constraints of the IRP.

2.1 Data Instances Description

The data instances used for the ROADEF/EURO 2016 Inventory Routing Problem (IRP) challenge were generated using a controlled random number generator (André et al., 2020). These instances were designed to mimic real-world operational conditions and included variations across several key parameters (Islam & Wasi, 2023):

i. Number of customers: ranging from 20 to 100.

ii. Number of vehicles: ranging from 5 to 20.

iii. Inventory demand: between 100 and 1000 units.

iv. Customer-to-customer distances: randomly generated.

v. Vehicle capacity: also generated randomly to introduce variability.

vi. Holding cost: a fixed cost applied per unit of inventory stored at each customer site.

vii. Backorder cost: a variable cost incurred when customer demand is not met on time.

In addition to these parameters, each instance adhered to several operational constraints:

i. All vehicles were required to start and end their routes at the depot.

ii. Each customer had to be visited exactly once within the planning horizon.

iii. The quantity delivered to each customer could not exceed the vehicle's capacity.

iv. Customer inventory levels needed to remain adequate to satisfy demand throughout the planning period.

To support the evaluation and ranking of participants, the organizers released the data in two sets:

- **Qualification Set:** This set contained 10 instances representing varying levels of complexity. Solutions submitted for these instances were used to rank participants during the preliminary phase.

- **Final Set:** A single, more challenging instance was provided for the final stage. This instance was specifically designed to assess the robustness, adaptability, and overall performance of each proposed algorithm.

The availability of these datasets played a crucial role in the challenge. They enabled participants to rigorously test their solution methods across diverse scenarios and compare their performance against competing approaches (Islam & Wasi, 2023). Moreover, the structure and variety of the instances helped reveal the strengths and weaknesses of different algorithms, as summarized in Table 1 below.

Table 1: Dataset A, B, and X Characteristics

Instance	Customers	Drivers	Trailers
A.1	2	-	-
A.2	3	-	-
A.3	3	-	-
A.4	4	-	-
A.6	3	-	-
A.7	12	-	-
A.8	12	-	-
A.9	12	-	-
A10	12	-	-
A11	12	-	-
V2.12	324	10	13
V2.13	53	10	5
V2.14	53	35	5
V2.15	134	10	4
V2.16	184	10	7

Instance	Customers	Drivers	Trailers
V2.17	134	35	4
V2.18	134	35	4
V2.19	53	35	5
V2.20	184	35	7
V2.21	184	35	7
V2.22	324	21	13
V2.23	324	21	13
V2.24	32	10	5
V2.25	32	35	5
V2.26	32	35	5
X.1	324	10	13
X.2	184	10	7
X.3	134	35	4
X.4	324	21	13
X.5	324	21	13

The attributes for each dataset are as follows:

Number of customers: The number of customers that need to be visited.

Number of vehicles: The number of vehicles that are available to make the deliveries.

3.0 Problem Formulation

3.1 Problem Notation

The key parameters, sets, and variables for the Inventory Routing Problem optimisation problem are captured by this notation (Islam, & Wasi, 2023).

Set: This constitutes time windows, trailers, base, source, and order express below**:**

i. TIMEWINDOWS(d): The range of times that driver d can be reached, from zero to T minutes in length.

ii. TRAILERS(d): The variety of trailers that Driver d can tow.

iii. Bases: Wherever and whenever changes begin and end.

iv. Sources and Customers: Loading and delivery locations, respectively.

v. ORDERS(p): The set of orders to be respected for customer p.

Parameter:

i. MAXDRIVINGDURATION(d): The longest amount of time that Driver d can be on duty for.

ii. MININTERSHIFTDURATION(d): The shortest possible period that driver d must wait in between shifts.

iii. LAYOVERDURATION(d): Driver d's layover time in hours.

iv. DISTANCECOST(tl): Trailer tl's unit price per kilometre travelled (in Euros).

v. CAPACITY(tl): The trailer's maximum weight in kilogrammes.

vi. INITIALQUANTITY(tl): The first usable product mass (in Kg) in the tl trailer at time zero.

vii. DISTMATRIX(p, q): The number of km separating points p and q.

viii. TIMEMATRIX(p, q): The number of minutes needed to get from point p to point q.

ix. SETUPTIME(p): How long it will take to load or deliver to location p (in minutes).

x. ALLOWEDTRAILERS(p): The pool of trailers that can transport goods to and from Customer P.

xi. CALLIN(p): Value 1 indicates a call-in customer, whereas Value 0 indicates a VMI customer.

xii. LAYOVERCUSTOMER(p): There must be a stopover between shifts if this is 1, else the customer can be serviced at any time.

xiii. SAFETYLEVEL(p): The level in the customer's tank must remain above the minimum required level at all times.

xiv. FORECAST(p, h): Customer p's product consumption in mass (Kg) at location p during timestep h.

xv. INITIALTANKQUANTITY(p): first Product Mass (Kg) at Horizon Start represents the first mass of product in the customer's tank.

xvi. MINIMUMOPERATIONQTY(p): The bare minimum of product that can be shipped in one go.

Variable:

i. arrival(o): Arrival time for operation o (in minutes).

ii. departure(o): Departure time for operation o (in minutes).

iii. layoverbefore(o): Indicates whether there is a layover before operation o.

iv. layoverafter(o): Indicates whether there is a layover after operation o.

v. orderQuantityFlexibility(o, p): The minimum ratio of the ordered quantity to deliver on customer p to consider order o satisfied.

vi. (od): Satisfaction level of order od.

vii. trailerQuantity(o): Quantity of product loaded or delivered during operation o (in Kg).

3.2 Decision Variable for Inventory Routing Problem

A solution to the problem consists of a set of shifts, denoted as SHIFTS. Within each shift s∈SHIFTS, we define the following decision variables (Islam, & Wasi, 2023):

- driver(s): Identifies the driver who is responsible for shift s.
- trailer(s): Identifies the trailer and tractor set for shift s.
- start(s): represents the beginning of shift s, expressed as a number of minutes between 0 and T.
- Operations(s): These are the tasks, such as loading and delivering, which occur during each shift.

Each operation o ∈ OPERATIONS(s) is characterized by the following attributes:

- arrival(o): Minutes inside the [0, T] range that represent the time of arrival for operation o.
- point(o): specifies whether o happens "at sources" or "at customers," both of which are possible options.
- quantity(o): Phase o involves defining the delivered or loaded quantity. Loading operations at sources are indicated by negative kilogramme values, whereas client deliveries are shown by positive kilogramme values.

It is not required that the number of hard and soft constraints be equal in order to find a realistic solution to the Inventory Routing Problem, as stated by Karaboga and Basturk (2007), Anagnostopoulos and Vrahatis (2011), and Rao and Savsani (2011). The opposite is usually true; more frequently than not, there will be more hard limits than soft ones. Here, the algorithm's goal is to discover a solution that not only meets the hard restrictions, but also to minimise the amount by which those constraints are broken (Islam, & Wasi, 2023). The constraints of the ROADEF/EURO 2016 Inventory Routing Problem are separated into hard and soft categories as follows:

3.3 Constraints Fomulation

In the ROADEF/EURO 2016 Inventory Routing Problem, the following constraints are considered:

Bounding constraints: These restrictions limit the availability of trucks and trailers, the volume of stock that can be moved, and the length of time that can be spent at each location.

$\forall$ X $\in= VARIABLES: unary\_constraints(X)$……………………………………… (1)

Where:

$\forall$: represents for each or for all

X: An individual variable from the set of VARIABLES

$unary_{constraints(X)}: The\ set\ unary\ constraints\ that\ must\ be\ satisfied\ for\ variable\ X.$

Constraints related to drivers: These constraints ensure that each driver does not work too many hours or drive too many kilometers.

For all drivers d and all consecutive shifts for S_1 and S_2 the constraint below must hold for driver d:

$\forall d \in DRIVERS, \forall S_1, S_2 \in shifts(d): Start(S_2) > End(S_1) + MININTERSHIFTDURATION(d)\ OR\ Start(S_1) > End(S_2) + MININTERSHIFTDURATION(d)$ ……………………………………… (2)

For a given shift s in SHIFTS, all operations o in the set of operations in a shift, the cumulated driving time is:

$$\forall s \in SHIFTS, \forall o \in \{Operations(s) \cup final(s)\}:$$

$$If\ LayoverBefore(o)\ then$$

$$cumulatedDrivingTime(o) = TimeMatrix(prev(o), o) - DrivingTimeBeforeLayover(o)$$

$$else$$

$cumulatedDrivingTime(o) = cumulatedDrivingTime(prev(o)) + TimeMatrix(prev(o), o$ ………………… (3)

The maximum allowed driving time constraints for each operation in shifts s is:

$$\forall s \in SHIFTS, \forall o \in \{Operations(s) \cup final(s)\}:$$

$cumulatedDrivingTime(o) \leq MaxDrivingDuration(driver(s))$ ……………. (4)

Each shift s must appear within individual time windows of the drivers selected:

$$\forall s \in SHIFTS:$$

$$\exists tw \in TIMEWINDOWS\big(Drivers(s)\big):$$

$Start(s) \geq Start(tw)\ and\ End(tw) \geq End(s)$…………… (5)

Constraints related to trailers: These constraints ensure that trailers are not overloaded and that they are compatible with the trucks that are used.

Avoiding overlapping; same trailer different shifts (S_1, S_2)

$$\forall tl \in TRAILERS, \forall S_1, S_2 \in shifts(tl):$$

$(start(S_2) > end(S_1))\ OR\ (start(S_1) > end(S_2))$ ………. (6)

The driver assigned to trailer must be compatible

$$\forall s \in SHIFTS:$$

$trailer(s) \in TRAILERS(driver(s))$ …………………. …… (7)

Constraints related to sites: These constraints ensure that deliveries are made to the correct sites and that the inventory is not damaged.

Considering customers tank capacity for each site (Not Call-In customer)

$$\forall p \in \{CUSTOMERS \mid CALLIN(p) = 0\}, \forall h \in [0, H[:$$

$0 \leq tankQuantity(p, h) \leq CAPACITY(p)$ …………….… (8)

Customers and sources dynamic equation

$$For\ all\ p \in \{CUSTOMERS \mid CALLIN(p) = 0\}:$$

$$tankQuantity(p, -1) = INITIALTANKQUANTITY(p)$$

$$For\ all\ h \in [0, H[:$$

$$dyn = tankQuantity(p, h-1) - FORECAST(p, h) + \sum o \in Operations(p, h)\ quantity(o)$$

$tankQuantity(p, h) = max(dy)$………………………… (9)

For each customer (excluding call-in customers) and source, this set of constraints defines the inventory dynamics, where the tank quantity at time step h is calculated by subtracting the consumption forecast at h from the sum of all deliveries made at h, and making sure the resulting tank quantity is non-negative. Each client's tank volume at the outset is predetermined.

Constraints related to shifts: These constraints ensure that deliveries are made during the allowed hours of operation.

Traveling from the previous point and the layover duration denotes arrival time express below:

$$For\ all\ o \in Operations(s), for\ all\ shifts\ s:$$

$arrival(o) \geq departure(prev(o)) + TIMEMATRIX(prev(o), o) + layoverbefore(o) * LayoverDuration(driver(s))$ ……………………………………………………… (10)

Loading and delivery as constant time:

$$For\ all\ Operations:$$

$departure(o) = arrival(o) + SetupTime(point(o))$ …………………………… (11)

Opening hours of customers are the delivery time of product:

$$For\ all\ shifts\ s\ in\ SHIFTS:$$

$$For\ all\ o \in Operations(s), there\ exists\ a\ time\ window\ tw\ in\ TIMEWINDOWS(point(o))\ such\ that:$$

$arrival(o) \geq start(tw)\ and\ end(tw) \geq departure(o)$…………………. (12)

Trailers accessing customers site:

$$For\ all\ shifts\ s\ in\ SHIFTS:$$

$$For\ all\ o \in Operations(s):$$

$if\ point(o) \in [CUSTOMERSUSOURCES]\ then\ trailer(s) \in ALLOWEDTRAILERS(point(o))$ …………………………………………….…… (13)

Trailer's quantity not exceeding it capacity:

$$For\ a\ given\ shift\ s\ in\ SHIFTS, for\ all\ o \in Operations(s)\ with\ \{final(s)\}:$$

$$trailerQuantity(o) = trailerQuantity(prev(o))$$

$$trailerQuantity(o) \geq 0$$

$trailerQuantity(o) \leq CAPACITY(trailer(s))$ ……………………………….… (14)

Following the previous shift, the initial quantity of trailer for a shift, is the end of quantity of trailer:

$$endTrailerQuantity(s) = trailerQuantity(last(Operations(s)))$$

$$If\ s = first(shifts(trailer(s))), then:$$

$$startTrailerQuantity(s) = INITIALQUANTITY(trailer(s))$$

$$else:$$

$startTrailerQuantity(s) = endTrailerQuantity(prev(s, shifts(trailer(s))))$………….…………………… (15)

Product must be loaded or delivered:

$$For\ all\ p \in CUSTOMERS, for\ all\ h \in [0, H-1], for\ all\ o \in Operations(p,h):$$

$$quantity(o) \geq 0$$

$$For\ all\ p \in SOURCES, for\ all\ h \in [0, H-1], for\ all\ o \in Operations(p,h):$$

$quantity(o) < 0$…………………………………………………………………… (16)

Capacity at customer site (not Call-In customer):

$$For\ all\ p\ \in\ \{CUSTOMERS\ |\ CALLIN(P)\ =\ 0\}, for\ all\ h\ \in\ [0, H-1], for\ all\ o\ \in\ Operations(p,h):$$

$$quantity(o)\ <=\ CAPACITY(p)$$

$quantity(o)\ >=\ MINIMUMOPERATIONQUANTITY(p)$ ……………………… (17)

This limitation guarantees that each VMI customer's delivered quantity is within their tank capacity and corresponds to the bare minimum needed for operations.

Constraints related to Layover: In the scheduling world, these are known as "soft constraints". A delivery window is the amount of time added to a shift to account for travel time to and from customers.

Layover after operation based on time duration:

$$For\ a\ given\ shift\ s\ \in\ Shifts, for\ any\ operation\ o\ \in\ \{\ Operations(s)\ \cup\ final(s)\}:$$

$$If\ (arrival(o)\ -\ departure(prev(o))\ >=\ LAYOVERDURATION(driver(s))\ +\ TIMEMATRIX(point(prev(o)), point(o))), then$$

$$layoverbefore(o)\ =\ 1$$

$$else$$

$layoverbefore(o)\ =\ 0$ ……………………… (18)

The variable layoverbefore (o) is defined in this constraint for each operation (o) in a shift (s), indicating whether or not there is a layover before the operation given the given length and travel timings.

If layover exist, shift must include layover:

$$For\ all\ shifts\ s\ \in\ Shifts:$$

$$If\ there\ is\ no\ customer\ p\ \in\ Customers(s)\ where\ LAYOVERCUSTOMER(p)\ =\ 1\ in\ Customers(s), then$$

$\sum o \in Operations(s)\ layoverbefore(o)\ +\ final(s)\ =\ 0$……………………… (19)

Only single layover per shift is permitted in each operation:

$$For\ all\ shifts\ s\ \in\ Shifts:$$

$\sum o \in Operations(s)\ layoverbefore(o)\ +\ final(s)\ \leq\ 1$……………………… (20)

This constraint enforces that only one layover is allowed per shift, ensuring that excessive layovers are not introduced in a single shift.

Constraints related to quality of service: Delivery delays and failure to fulfil customer expectations will result in penalties under these limits. The overall distance travelled or the number of deliveries are to be made on time which determine the solution's quality (Islam, & Wasi, 2023).

Call-in customer order to be satisfied in at least one operation:

$$For\ all\ p\ \in\ \{c\ \in\ CUSTOMERS\ |\ CALLIN(c)\ =\ 1\}, for\ all\ od\ \in\ ORDERS(p):$$

$orderQuantityFlexibility(od) * quantity \leq \sum_{op \in \{operation(s) | earliastTime(od) \leq arrival(op) and\ arrival(op) \leq arrivalTime(od), s\ shifts\}} Quantity(op) \leq quantity(od)$…………………………………………………………………… (21)

Each call-in customer order must be fulfilled by one or more actions, and these operations must commence after the earliest time and before the latest time specified in the order to satisfy this constraint.

For call-in customer operations are related to order:

$$For\ all\ shifts\ s, for\ all\ operations\ op\ \in\ \{operation(s)\}:$$

$$IF\ POINT(op)\ \in\ \{p\ \in\ CUSTOMERS\ |\ CALLIN(p)\ =\ 1\}, THEN$$

$There\ exists\ an\ order\ od\ \in\ ORDERS(POINT(op))\ such\ that\ arrival(op)\ \geq\ earliestTime(od)\ and\ latestTime(od)\ \geq\ arrival(op).$ ……………………… (22)

Avoid run-out:

$$For\ all\ p\ \in\ CUSTOMERS\ |\ CALLIN(p)\ =\ 0, for\ all\ h\ \in\ [0, H-1]:$$

$SafetyLevel(p)\ \leq\ tankQuantity(p,h)$ ………………………………… (23)

VMI clients are constrained by this rule to maintain inventory at or above the minimum safe levels at all times.

Whether a constraint is considered hard or soft is context dependent. The goal is to minimize the distribution costs (D) required to meet customer demands for the product over the long term horizon.

$$let;\ \mathrm{D} = \text{the distribution cost over the long term horizon.}$$

$$\mathrm{Q} = \text{the total quantity delivered on shifts.}$$

$$\mathrm{C(s)} = \text{the total cost of shift s.}$$

$$\mathrm{LR(s)} = \text{the logistic ratio for shift s}$$

In order to achieve this, we will minimize the logistic ratio (LR) defined as the total cost of the shifts (C(s)) divided by the total quantity delivered on those shifts (Q): $\mathrm{D} = \min\{\mathrm{LR(s)}\}$ for all shifts s

$LR(s) = \sum\left(\frac{C(s)}{Q}\right)$……………………… (22)

4.0 Methodology Approach of IRP

The progression is illustrated in Figure 1 below. The review begins with the primary research strategies, which include exact-method algorithms, such as the Branch-and-Cut technique (a), and approaches grounded in Game Theory (b). Moving to the next level of classification, the study considers both Local Search methods (a) and Population-Based methods (b). These approaches share several conceptual similarities, which is why they are collectively grouped under Metaheuristics.

The final category emphasized in Figure 1 is the Hyper-heuristic approach, underscoring its growing relevance and importance within the field. This layered categorization reflects the diverse techniques, tools, and methodological perspectives employed across the reviewed studies, highlighting the multifaceted nature of the research landscape.

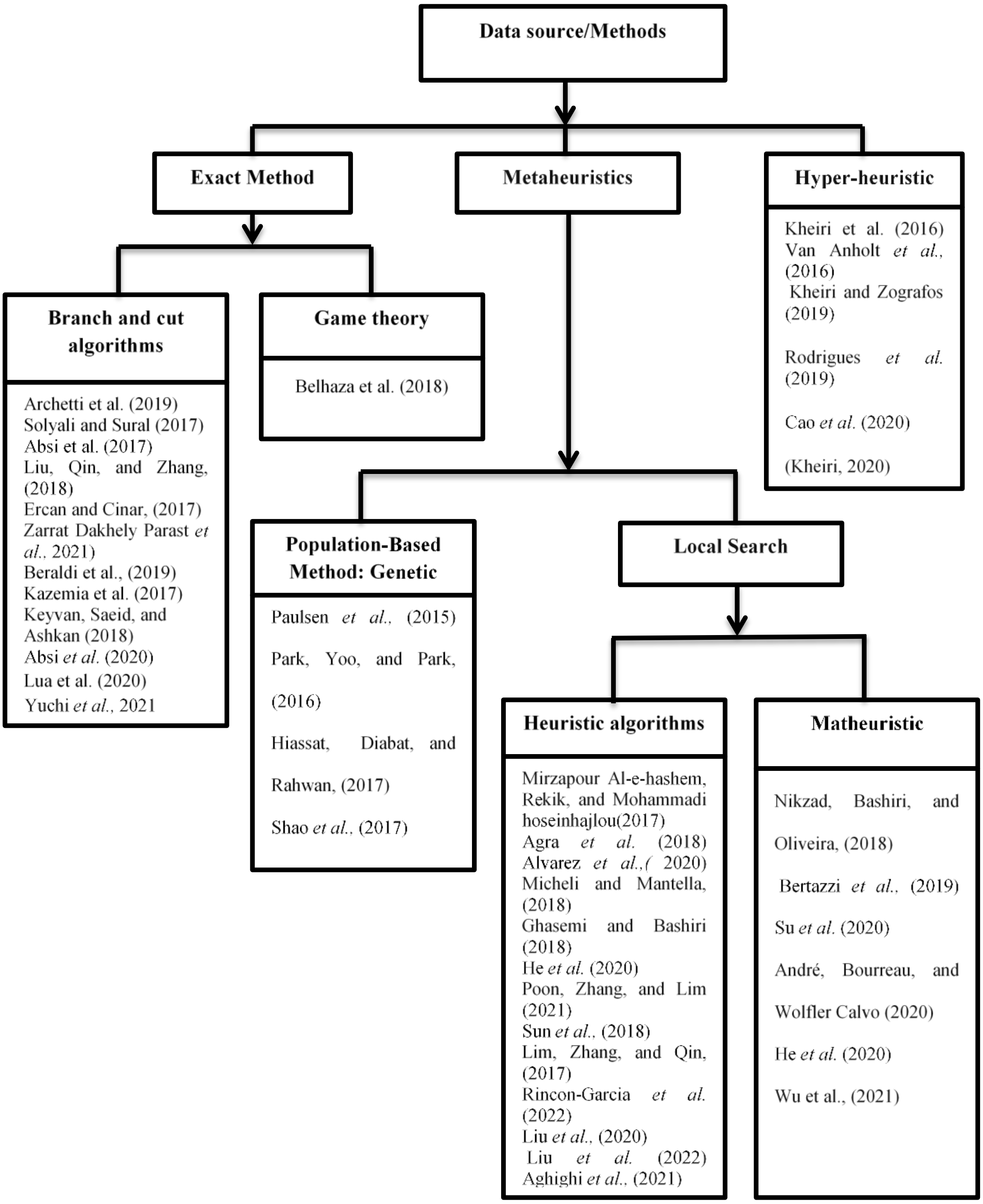

Figure 1: IRP Methodology approaches

4.1 Exact Method:

4.1.1 Branch and cut algorithms

This algorithm offers an effective means of solving a broad spectrum of integer programming problems by integrating insights from multiple approaches to produce a workable solution. In their study, Solyali and Sural (2017) first examined the limitations of the existing formulations and then developed a strong formulation (SF). They dynamically incorporated the SF constraints and paired them with separation algorithms to ensure full compliance with the problem's rules and conditions. Their approach is particularly valuable in situations exhibiting polynomial characteristics.

The matheuristic technique employed by Archetti et al. (2019) extends this line of research by generating columns through pricing based on routing decisions and customer lists within specific time horizons. Drawing from branching, pricing, and cutting tree concepts, they developed a hybrid method that merges column generation with cutting-plane techniques. Their approach uses an undirected graph in which all edges initially carry no connections, and every node represents a location either a customer or a depot. The pricing mechanism adheres to the triangle inequality. The planning horizon is defined in days and incorporates customer capacity limits and the total volume to be dispatched from the depot. Additional constraints, such as vehicle capacity, prevention of overlaying operations, and maximum customer inventory levels, were all integrated to formulate an optimization strategy aimed at minimizing the logistics ratio.

In a related contribution, Absi et al. (2017) introduced a parameter z to linearize the objective function, albeit in a slightly different modelling context. To address issues in the initial set of shifts, they proposed time-stamped shifts and examined the generated solutions to identify any non-zero artificial variables that could be aligned with feasible outcomes. A heuristic pricing procedure was then applied to generate low-cost new shifts (Liu et al., 2018). This process was iteratively refined until satisfactory results were obtained.

Addressing sustainability concerns, Ercan and Cinar (2017) examined green logistics within the broader supply chain literature. Their study focuses on a multi-product, multi-vehicle IRP in which rising fuel costs are considered a critical sustainability factor. Their formulation incorporates demand uncertainty and the opportunity cost of holding inventory. To tackle this complex problem, they adopted a sample average approximation method. As emphasized by several researchers (Zarrat et al., 2021), the overarching objective is to minimize total operational costs, including transportation and inventory expenses. Because transportation costs include fuel consumption, the environmental implications of fuel usage cannot be overlooked. Fuel usage is carefully calculated using variables such as vehicle load, travel distance, and technical specifications (Islam & Wasi, 2023). Their stochastic model involves a first stage that determines routing decisions, followed by a second stage that determines shipment quantities and inventory levels. Using datasets from existing literature, they evaluated the method based on computational time and optimality gaps. The findings were promising and highlight potential avenues for future research in this rapidly evolving domain (Beraldi et al., 2019).

In another application area, Kazemia et al. (2017) explored the Perishable Inventory Routing Problem (PIRP) by addressing the highly sensitive task of transporting blood products from regional blood centres to hospital blood banks. Ensuring timely delivery is essential, not only to avoid spoilage but also to minimize storage and transportation costs. Unlike typical PIRPs, blood supply chains involve complex considerations such as allocated versus unallocated stock, cross match release times, cross match-to-transfusion ratios, and an “older-first” usage policy. These features make conventional PIRP models insufficient. To address this gap, the authors developed a mixed integer programming framework tailored to blood logistics and proposed a robust possibilistic programming (RPP) approach to handle system uncertainties. To efficiently identify global optimal solutions, they employed a new iterative branch-and-cut method across a variety of numerical experiments. A case study using data from the Iranian blood supply chain demonstrated the practical value of their model and the robustness of the RPP-based solution approach.

To deepen the understanding of how collaboration among logistics providers enhances upstream supply chain performance, Keyvan et al. (2018) conducted an extensive investigation. In their study, logistics providers are initially treated as independent entities. A mixed-integer programming (MIP) formulation is then introduced to model the Cooperative Inventory Routing Problem (Co-IRP), enabling the analysis of various coalition structures and allowing researchers to measure collective efficiency under different cooperative settings. Each logistics provider operates a depot and a set of retail outlets that jointly constitute their market. Once a coalition is formed, the problem is reformulated into a many-to-many network structure, facilitating product movement from multiple depots to multiple stores more effectively. To address uncertainties at the network level, the authors extend the model to a more robust version known as Co-RIRP. The study also evaluates several payoff-allocation mechanisms to ensure that cost savings are distributed fairly among coalition members, offering valuable insights through numerical results and managerial interpretations.

Similarly, Absi et al. (2020) employed a master-problem approximation combined with a branch-and-cut heuristic for pricing within the context of the ROADEF/EURO 2016 Challenge. This hybrid approach successfully generated feasible solutions. The authors enhanced the objective function by incorporating early-delivery considerations to better detect practical solutions. By integrating a delivery profit vector into the linearized objective function together with valid inequalities and artificial variables the study demonstrated how branch-and-cut heuristics can support profit maximization (Desaulniers et al., 2016).

A different direction of research is presented by Lua et al. (2020), who investigated the Time-Dependent Electric Vehicle Routing Problem (TDEVRP). This problem requires determining optimal routes for a fleet of electric vehicles while jointly optimizing vehicle speeds and departure times for each segment of the route. The authors formalized the TDEVRP using an Integer Linear Programming (ILP) model; however, ILP solutions via CPLEX are only computationally feasible for instances with fewer than 15 customers. To tackle larger cases, they developed an Iterated Variable Neighbourhood Search (IVNS) algorithm. This approach integrates:

- Variable Neighbourhood Descent (VND) to optimize node ordering,
- A Departure Time and Speed Optimization Procedure (DSOP) to fine-tune speeds and departure times, and
- A constant-time evaluation method (O(1)) to efficiently assess local search moves.

The IVNS algorithm demonstrated exceptional performance, consistently producing optimal or near-optimal solutions for small instances and maintaining robustness for larger ones by reducing the average optimality gap (Yuchi et al., 2021). Further experiments on the Time-Dependent Pollution Routing

Problem (TDPRP) also confirmed the strength of IVNS, with the algorithm even establishing new best-known solutions (Mara et al., 2021).

4.1.2 Game theory.

Belhaza et al. (2018) applied this concept within the context of routing challenges. In their approach, the algorithm receives inputs that hold strategic value for all participating agents, and this information is shared among them. Rather than limiting their focus to travel duration alone, the researchers considered multiple criteria aimed at reducing trip time while simultaneously increasing value for both customers and drivers.

Their methodology aligns with several principles of game theory, particularly in the search for a Nash Equilibrium. To achieve this, they integrated a hybrid neighbourhood tabu search technique, enabling the identification of Pareto-efficient solutions those not dominated by any alternative options.

After defining the structure of vehicle routing problems, the authors reframed these problems as multi-agent non-cooperative games. This allowed them to incorporate factors such as customer demand quantities and the total number of deliveries. They further outlined the customers' optimal decision-making strategy based on Nash equilibrium principles and provided the specific conditions under which such equilibrium could be attained.

4.2 Metaheuristics

In order to efficiently explore the space of possible solutions, researchers often employ metaheuristics strategy (Fatemi et al., 2019) in obtaining significant results. This strategy is divided into two distinct categories: local search strategies and population-based search strategies.

4.2.1 Local Search Method

Local search strategies concentrate entirely on improving the current solution state, rather than exploring multiple pathways simultaneously (Karakostas et al., 2019). This approach underlies many heuristic and matheuristic techniques, which guide search processes through local neighbourhoods to find better solutions.

a. Heuristic Algorithms

Mirzapour et al. (2017) highlight the growing need for environmentally friendly logistics, particularly given the uncertainty in customer demand. They proposed a "green" transshipment-enabled framework for the Inventory Routing Problem (IRP) within a many-to-one distribution context. Their model adopts a two-stage stochastic programming approach with dual objectives:

(1) minimising supply chain costs, including shortage penalties, and

(2) reducing greenhouse gas (GHG) emissions from transport and waste management.

The study introduces transshipment as a key mechanism to reduce transport costs, curb emissions, and manage uncertainty. A hybrid algorithm combining the L-shaped decomposition technique and compromise programming is employed to address this multi-objective model. The findings illustrate how firms can strike a sustainable balance between economic performance and environmental responsibility, demonstrating the strategic value of transshipment in modern green supply chains.

Agra et al. (2018) investigate a maritime IRP involving a single product with stable production and consumption levels over a fixed planning horizon. The complexity arises from heterogeneous fleets, numerous ports, and limited storage capacity. Shipping times are heavily affected by weather unpredictability. To handle this uncertainty, the researchers adopt a flexible robust optimisation framework using the budget polytope uncertainty set. While routing decisions, visit sequences, and loading volumes are predetermined, visit timings and inventory levels can adapt to unfolding scenarios (Alvarez et al., 2020).

Their solution incorporates a decomposition strategy alternating between a master problem and an adversarial separation problem that identifies infeasible scenarios. Additional algorithmic enhancements are proposed to reduce runtime and iteration count (Turky et al., 2023). A recursive local search heuristic further refines the approach. Computational tests using real-life maritime data confirm the practicality and efficiency of their techniques.

Micheli and Mantella (2018) examine how transportation-related activities contribute to carbon emissions and how emerging carbon regulation policies may influence supply chain decisions. Unlike traditional IRP models that optimise routes based on stock levels, this study integrates uncertain customer demand, detailed emissions modelling, and diverse vehicle types. Four policy scenarios are evaluated: carbon cap, carbon tax, cap-and-trade, and cap-and-offset. Using a probabilistic mixed-integer linear programming (MILP) model, the authors assess both the financial and environmental implications of each policy. Their results help managers understand how regulatory choices affect overall supply chain sustainability and cost performance.

Ghasemi and Bashiri (2018) introduce a new model for managing long-term blood collection and distribution in healthcare supply chains. Their framework handles periodic stochastic demand and incorporates both stationary blood centres and mobile "bloodmobiles," which operate without needing to return to the depot after each shift. A dedicated vehicle transports collected blood from mobile units to the central facility, following the guidelines of the Iranian Blood Transfusion Organisation (IBTO).

To address the perishable nature of blood and the uncertainty in supply, the authors propose a two-stage stochastic selective-covering-inventory-routing (SCIR) model. In the first stage, the number of bloodmobiles is determined before actual scenarios unfold. In the second stage, costs—including travel time, trailer usage, and driver waiting—are minimised according to a logistic ratio framework (He et al., 2020). This model provides a robust solution for managing volatile medical supply chains.

Keskin, Laporte, and Catay (2019) focus on the Electric Vehicle Routing Problem with Time Windows (EVRPTW), where limited vehicle range and charging station congestion create additional complexity. They propose a mixed-integer linear programming model that captures time-dependent queues, penalising late arrivals and rewarding timely scheduling. To solve larger instances, the authors develop matheuristics combining Adaptive Large Neighbourhood Search (ALNS) with mixed-integer subproblem solving. Their results show that queue-induced delays significantly influence charging patterns, often shifting recharging activities to less congested mid-day periods.

Poon et al. (2021) examine urban transport routing under time-varying travel speeds, repeated vehicle trips, and slow depot loading times. Their objective is to minimise total travel distance while adhering to load limits, time windows, and trip duration constraints. Sun et al. (2018) formulate this as the Multi-Trip Time-Dependent VRP with Time Windows (MT-TDVRPTW). They develop piecewise linear travel-time approximations within a segment-based evaluation framework to speed up neighbourhood search. Building on this foundation, Lim et al. (2017) construct a hybrid metaheuristic combining ALNS for broad exploration and VND for deep local refinement. Extensive experiments show strong performance across diverse problem settings, demonstrating the algorithm's robustness under fluctuating traffic conditions and operational constraints.

Rincon-Garcia et al. (2022) emphasise the increasing importance of efficient urban delivery systems in response to new retail trends. Urban congestion and driving-hour regulations significantly affect routing efficiency. Responding to this, Liu et al. (2020) introduce a large neighbourhood search algorithm for the time-dependent VRP that incorporates EU driving-hour rules (EC 561/2006). Their approach outperforms benchmark heuristics by reducing fleet size, travel distance, and duty time. The method also supports scenario-based analysis for home delivery operations, enabling assessments of customer density, congestion, time window policies, and compliance effects. André et al. (2020) note that such integrated frameworks capture the multifaceted challenges of modern urban logistics.

Liu et al. (2022) review traditional logistics decision models involving distribution centre placement, inventory control, vehicle scheduling, and routing optimisation. Research has addressed multi-depot, multi-echelon, and multi-period multi-product problems, with a strong focus on minimising supply chain costs (Aghighi et al., 2021). Multi-objective optimisation, replenishment strategies, and identical vehicle fleets are widely studied, particularly for perishable goods logistics. Stochastic modelling plays an increasingly critical role due to the prevalence of uncertain operating environments. Across the literature, heuristic methods remain a dominant approach for solving the complex Logistics Inventory Routing Problem (LIRP).

b. Matheuristic Algorithms

Nikzad et al. (2018) study demand uncertainty using a two-stage stochastic model that incorporates random constraints. They propose a matheuristic method—combining heuristic principles with mathematical programming—and employ Latin hypercube sampling to generate representative scenarios. Numerical experiments validate the importance of accounting for stochasticity and confirm the effectiveness of their solution approach. Similarly, Bertazzi et al. (2019) show that properly modelling uncertainty is vital for achieving realistic and reliable IRP solutions.

Su et al. (2020), winners of third place in the ROADEF/EURO 2016 Challenge, develop a matheuristic that integrates local search with mathematical programming to meet competitive scoring requirements. Their method explores six neighbourhood structures using operators that add or remove shifts to refine routing decisions. A mixed-integer programming model handles timing constraints, while a linear programming model deals with continuous quantity variables (Wu et al., 2021). As summarised by André et al. (2020), the method decomposes the problem into two subcomponents timing and quantity each addressed using appropriate optimisation tools.

Another influential matheuristic is presented by He et al. (2020) in "A Matheuristic with Fixed-Sequence Reoptimization for a Real-Life Inventory Routing Problem." The authors generate candidate routes using two greedy algorithms and a column-generation heuristic, with each column representing a timed route. Their fixed-sequence mixed-integer linear fractional programming model is solved using the Dinkelbach method, extending the divide between exact and heuristic methods in terms of computation and scalability. Notably, none of the reviewed matheuristics provides formal lower bounds, highlighting an ongoing research challenge (André et al., 2020).

He et al. (2020) further refine matheuristic approaches by incorporating unique characteristics of real-world inventory routing instances. Their fixed-sequence mathematical programme was instrumental in obtaining top-ranking solutions in the ROADEF/EURO 2016 Challenge, successfully optimising delivery schedules and loading plans within the competition's strict time limits.

4.2.2 Population-Based Method

Population-based methods emphasize that relying on a single solution node is not sufficient for solving large-scale routing problems. Instead, these methods evaluate interactions among multiple nodes (locations) within the search space to guide the population toward better global solutions.

Coelho et al. (2020) examine the Multi-Attribute Inventory Routing Problem (MAIRP), one of the more complex challenges in supply chain management. They offer a detailed mathematical formulation and propose exact solution strategies. Building on the multi-product context, Bertazzi et al. (2017) extend the Multi-Depot Inventory Routing Problem (MDIRP) by incorporating heterogeneous vehicle fleets and explicit constraints on route duration (Guimarães et al., 2019). The MAIRP is NP-hard because it integrates several real-world characteristics from vendor-managed inventory systems, making it more complex than the classical IRP.

To tackle this complexity, the authors develop a hybrid exact algorithm in which multiple Mixed Integer Programming (MIP) models are solved within a customised Variable Neighbourhood Search (VNS) framework (Van Anholt et al., 2016). This hybrid structure significantly accelerates convergence in comparison to the traditional branch-and-cut procedure applied directly to a standard MIP model. Extensive computational experiments on well-known benchmark datasets and real-world instances—both newly generated and taken from prior research—demonstrate the strong efficiency of the proposed method in generating high-quality, practical solutions (Laganà et al., 2016).

The following population-based method is commonly applied:

a. Genetic Algorithms (GAs)

Genetic algorithms operate by selecting the fittest individuals in a population to reproduce and generate new candidate solutions for subsequent generations (Park et al., 2016). Paulsen et al. (2015) applied a heuristic procedure as an initial step, followed by a GA to address complex constraints within their routing model. Their framework is adaptable, performing well in cases where upper bounds are known or when working with small-scale instances that can be solved with exact methods.

A critical component of their approach is the incorporation of time windows for each customer node. They assigned multiple interval states and applied a GA to manage these time-dependent constraints (Hiassat et al., 2017). Additionally, they introduced a dynamic programming strategy to determine the optimal time span for a Travelling Salesperson to visit the required stops. This step is essential for allocating customers to vehicles before running the GA. Through this integration of heuristics and genetic search, Paulsen et al. (2015) aimed to enhance the efficiency of solving complex vehicle routing problems, particularly those that involve strict time-window constraints.

Electric vehicle routing introduces additional difficulties, which Shao et al. (2017) addressed by designing an algorithm that incorporates charging times and varying travel durations. They used a GA to find the optimal combination of travel time, stop locations, and charging fees. To determine the shortest paths between nodes, the model also employed a dynamic Dijkstra algorithm. Time-dependent travel durations were included to reflect real traffic variations. The effectiveness of the proposed approach was validated through a large-scale case study in Beijing, confirming the practical feasibility of the model and its solution strategy.

4.3 Hyper-heuristic

Hyper-heuristics automate the process of selecting, combining, adapting, and generating multiple simple heuristics to improve optimisation performance (Turky et al., 2023). Kheiri and Zografos (2019) used a sequence-based hyper-heuristic to prevent the search process from getting trapped in local optima. Their approach performs an extensive analysis during optimisation and dynamically constructs sequences of heuristics that are more likely to guide the search toward high-quality solutions.

Building on this idea, Kheiri et al. (2016) introduced sequence-selection hyper-heuristics that choose and apply low-level heuristics from a predefined set to candidate solutions. To support adaptive decision-making, they developed a Markov model in which low-level heuristics act as hidden states. As the optimisation progresses, the model learns which heuristics—or combinations of heuristics—are most effective and increasingly prioritises them. This adaptive behaviour helped the hyper-heuristic outperform several competing methods during the ROADEF/EURO Challenge (Kheiri, 2020).

Using maritime transportation as a practical example, Rodrigues et al. (2019) analyse the effect of uncertainty in planning environments where factors such as port congestion and weather conditions are highly unpredictable. They examine a marine inventory routing problem involving a single product, fixed production and consumption rates, a heterogeneous fleet, and multiple ports. The study adopts a two-stage decision-making structure. In the first stage, all route-related decisions—such as the order of port visits and loading/unloading quantities—are predetermined (Van Anholt et al., 2016). In the second stage, once travel times become known, adjustments are made to port schedules and inventory levels to reflect the updated conditions.

Cao et al. (2020) provide a comprehensive review of recent advances in the Inventory Routing Problem (IRP). Their study categorizes IRP variants such as one-to-one models, one-to-many structures with tree-shaped routing patterns, and infinite-horizon IRP settings with stochastic demand. They also classify the major modelling approaches and solution techniques that have appeared in the IRP literature. Furthermore, the authors highlight the algorithms most frequently applied in recent IRP studies and identify promising directions for future research.

4.4 Ranking Overview Methods:

Air Liquide an international leader in industrial and medical gases, technologies, and related solutions presented the **ROADEF/EURO 2016 Challenge** as a real-world optimisation problem (Artigues et al., 2018). The central aim of this competition was to design an innovative approach capable of **reducing**

the per-unit cost of bulk gas deliveries (Nikzad et al., 2018). Achieving this objective required addressing a complex set of operational constraints, including:

i. Managing a single product type.

ii. Coordinating a heterogeneous fleet of vehicles.

iii. Working with drivers who differ widely in behaviour and skill.

iv. Operating across multiple planning horizons.Forecasting consumption using a deterministic demand model.

v. Adhering to strict delivery windows defined by tight scheduling rules.

In essence, the challenge demanded an optimisation methodology that could **holistically integrate these constraints** while ensuring substantial reductions in delivery costs (Keyvan et al., 2018; Poon et al., 2021).

A total of **35 teams** participated in the ROADEF/EURO 2016 Challenge, each proposing a solution to this large-scale industrial IRP. Their submissions underwent rigorous evaluation based on several core criteria (Rincon-Garcia et al., 2022):

1. Solution Quality

Teams were assessed on both the **thoroughness** and **originality** of their approaches. The primary evaluation metric was the **total delivery cost** achieved within the required time frames. Solutions with lower operating costs were considered more competitive (Rincon-Garcia et al., 2022).

2. Robustness

The robustness of each solution was evaluated by examining how well it performed under **perturbed or modified input data**. A robust approach maintained high performance despite variability or uncertainty in operational conditions (Nikzad et al., 2018).

3. Scalability

Scalability testing focused on the ability of each method to handle **larger and more complex problem instances**. Effective solutions demonstrated consistent performance without major delays or degradation as problem size increased (Artigues et al., 2018).

4.4.1 Review Process Overview

Candidate Qualification Stage

In the first phase, all submitted solutions were tested using a common set of benchmark instances. The purpose was to identify the most promising approaches based on performance consistency and computational efficiency. From this initial screening, the top ten teams advanced to the final round of the competition.

Visual Representation of Participation (2016–2023)

Your message includes a bar chart illustrating participation trends in the ROADEF/EURO IRP challenge from 2016 to 2023 (Nikzad et al., 2018). In the chart:

- Blue bars: These represent the authors or teams.
- Bars positioned left of the red dotted line correspond to teams that advanced to the final round.
- Bars positioned to the right represent teams that did not progress.

This figure 2 graphical depiction helps distinguish competitive solution providers from those that fell short during the qualification stage, highlighting the evolving interest and contributions to IRP research in the years following the challenge.

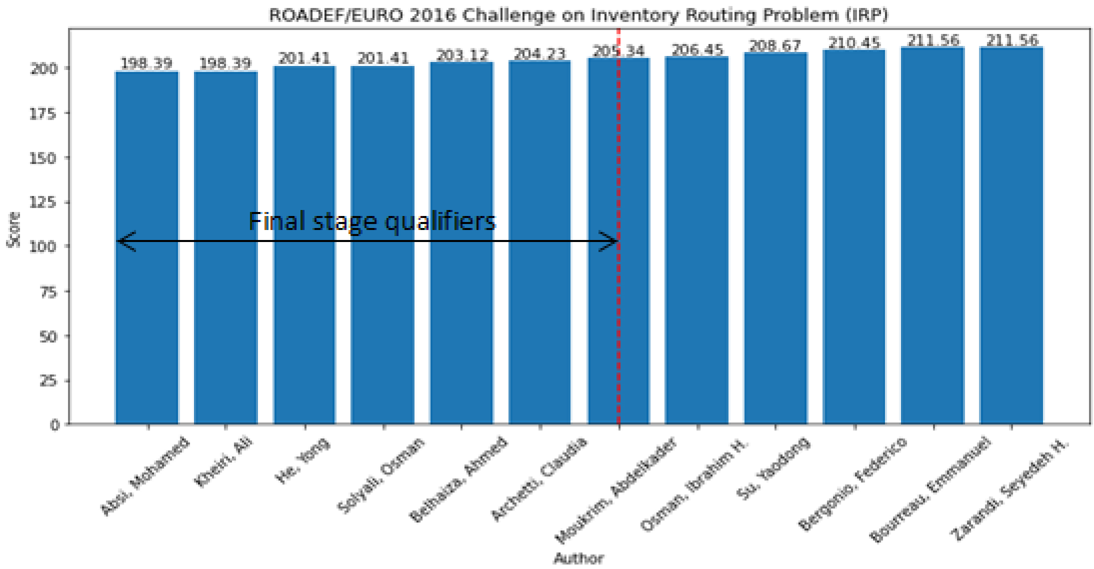

Figure 2: Participants qualification stage

Final Phase Challenge

In the final stage of the competition, the **ten qualified teams** as illustrated in Figure 3 were presented with a new and deliberately more complex set of problem instances. These cases were designed to push the limits of each team's methodology and test the adaptability of their proposed optimization strategies. The teams were tasked with developing solutions capable of addressing these heightened challenges.

As in the earlier phase, each final-round submission was evaluated using the same three core criteria: **solution quality**, **robustness**, and **scalability**. This allowed for a fair comparison of performance and ensured that only the most effective and resilient approaches were distinguished in the final rankings.

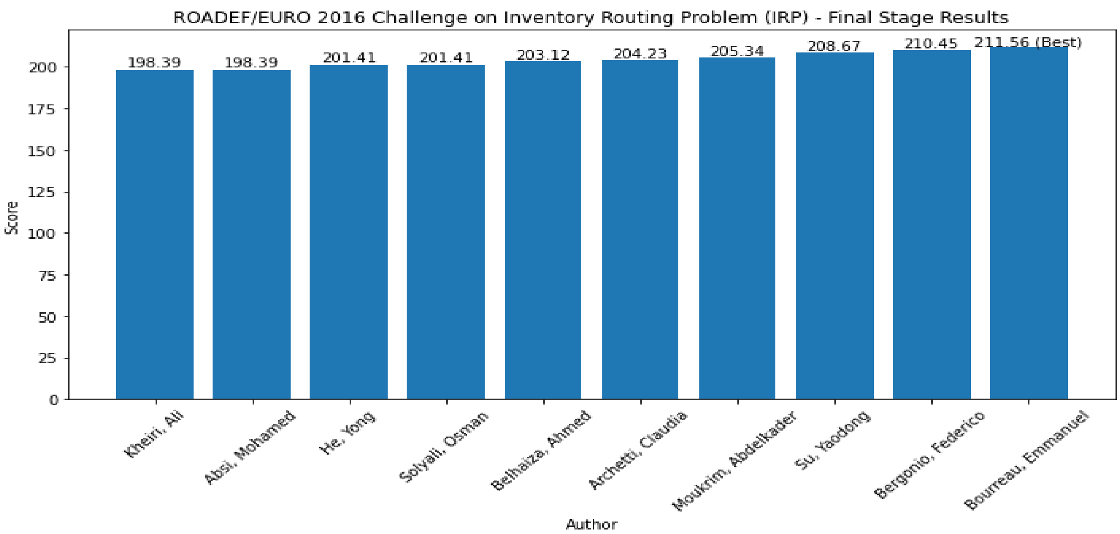

Figure 3: Final phase of the competition

Final Ranking Determination

The final rankings illustrated in Figure 3 were determined by assessing how effectively each of the ten finalist teams handled the advanced problem instances presented in the last phase. Ultimately, the winner of the ROADEF/EURO 2016 Challenge was the team whose solution demonstrated the strongest balance of solution quality**,** robustness**, and** scalability (André et al., 2020).

The team from the University of Technology of Compiègne (UTC), France**,** emerged as the overall winner. According to Moukrim et al. (2019), their success was driven by the adoption of a matheuristic algorithm a hybrid technique that strategically combines the strengths of local-search metaheuristics with mathematical programming. This innovative blend enabled them to navigate the complex constraints of the challenge more efficiently than competing teams, securing their top position.

Figure 4 presents a bar chart showing participation in the ROADEF/EURO Inventory Routing Problem (IRP) Challenge from 2016 to 2023. The y-axis shows the number of submissions, while the x-axis indicates the corresponding years. The data reveal that while the 2016 edition attracted 10 submissions, participation dropped sharply to one submission per year between 2017 and 2020. No submissions were recorded in 2021 or 2022, and only a single submission has been documented as of 2023.

This steady decline in participation may reflect the growing difficulty of developing significantly improved algorithms for the IRP, especially as existing approaches have become highly sophisticated. Nevertheless, the lone submission in 2023 signals that interest in the IRP challenge still persists, albeit at a smaller scale. Considering the IRP's far-reaching implications for transportation, logistics, and manufacturing sectors, continued exploration of innovative optimisation approaches remains highly relevant.

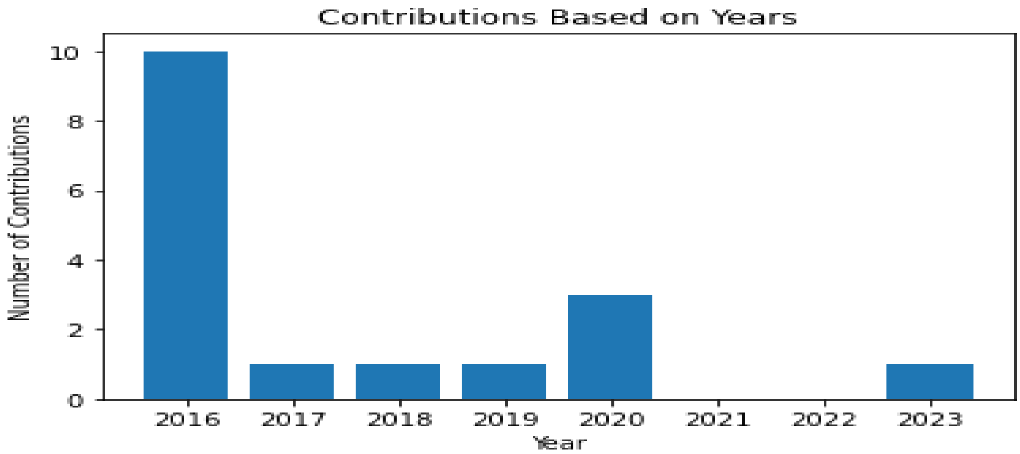

Figure 4: Contributions based on years by authors

Citations like *Kheiri et al. (2016)* identify the research sources where these strategies were first proposed or explained.

Therefore, sets B and X follow a similar structure, listing additional methods, their performance indicators, and supporting references. Together, these sets provide a useful benchmark for both researchers and industry practitioners, enabling them to compare and assess the effectiveness of various optimisation strategies developed for the IRP (Ercan & Cinar, 2017).

A critical review of these solution strategies reveals several limitations:

- **Hyper-heuristics**: While Hyper-heuristics is flexible, it may struggle when dealing with many decision parameters. Their overall effectiveness is not always guaranteed, particularly in highly complex search spaces (Turky et al., 2023).

- **Heuristic methods**: Often heuristic method require extensive computational time. They may need numerous iterations to identify all neighbourhood restrictions or "tabu" moves, making them slower to converge.

- The **logistic ratio**: Logistic ratio used in some IRP-related research, can have limited value because it does not always exhibit a strong or linear relationship with problem characteristics.

- **Branch-and-cut heuristics**: Branch-and-cut heuristic tend to be time-intensive to implement and may not scale efficiently when applied to very large IRP instances.

- **Metaheuristic approaches**: Although metaheuristic approach is powerful, can also be computationally expensive in practice. They may require long run-times or fine-tuning to achieve optimal performance (Kheiri et al., 2016).

5.0 Conclusion

In summary, research on the Inventory Routing Problem (IRP) has expanded significantly in recent years, with solution strategies becoming increasingly diverse. Scholars now categorize IRP methods under labels such as *hyper-heuristic*, *heuristic*, *metaheuristic*, and others. These classifications help both researchers and practitioners navigate the broad spectrum of tools available for tackling this complex optimisation problem.

However, evaluating these approaches has also revealed several limitations. As noted by Turky et al. (2023), hyper-heuristics can struggle when dealing with highly complex parameter sets or large-scale datasets. Heuristic methods, although practical, often require substantial time to implement, especially when searching through extensive neighbourhoods to identify tabu moves. The logistic ratio, used in some IRP studies, may also offer limited insight due to its weak or non-linear relationship with key system characteristics. In addition, branch-and-cut heuristics can be computationally demanding, and metaheuristics, despite their power, may require long processing times when applied in real-world scenarios.

Yet, these challenges highlight not hinder the field's evolution. The continued refinement of IRP models demonstrates growing attention to real-world complexities such as probabilistic demand, contextual factors, operational constraints, and sustainability considerations. These improvements are essential for enabling more economically viable and environmentally responsible supply chain decisions (Archetti et al., 2019).

Looking ahead, further progress depends on advancing algorithm design, strengthening modelling methodologies, and enhancing problem-solving strategies capable of addressing the multifaceted nature of inventory routing (Kheiri & Zografos, 2019). Encouragingly, the adoption of robust optimisation techniques, population-based methods, and cooperative or multi-agent approaches shows that meaningful innovation is already underway.

Ultimately, inventory routing remains is a vital research area within operations research. As global supply chains continue to grow in complexity, scholars and practitioners alike are committed to discovering new, efficient, and sustainable ways to improve routing performance. The ongoing development of advanced IRP strategies reflects this broader goal of closing the gap between theoretical innovation and practical application.

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