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# Fractional Factorial Design on Crop Yield Using Different Fertilizer Combinations

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**ABSTRACT**

Confounding occurs when you have a fractional factorial design and one or more factor effects that cannot be estimated separately. In this paper, we construct a $3^{4-1}$ fractional factorial design by partitioning the $3^4$ factorial design into three blocks, each with 27 treatment combinations. One of these one-third fractions was then used for the analysis of a $3^{4-1}$ fractional factorial design, and also used in testing for significance of the factor effects. The four factors considered were: Nitrogen, Phosphorus, Potassium and Manganese. From our analysis, the main effects Nitrogen, Potassium, Manganese was significant, the interactions: Nitrogen and Phosphorus, Nitrogen and Potassium, were also seen to be significant, while other factors were not significant.

Keywords: Confounding, Defining Relation, Resolution, Fractional Factorial Design, Blocking, Replication.

## 1. INTRODUCTION

### *1.1 Background of the study*

Design of experiment is the process of planning an experiment so that appropriate collected data that can be analyzed by statistical methods, resulting in a valid and objective conclusion [7]. It is a plan for assigning experimental units to treatment levels alongside the statistical analysis associated with the plan [2; 7; 14]. The design and analysis of experiments revolves around an understanding of the effects of different variables on other variables. It establishes a cause-and-effect relationship between a number of independent variables (or factors) and the dependent variables (response) of interest [4].

Design of experiment is divided into four (4) broad categories or types: reliability design of experiment, response surface methodology, one-factor design, and factorial design. Among these categories of experimental designs, the factorial design was fully explored. [7] defined factorial experiments as experiments in which each complete trial or replication of the experiment and all possible combinations of the level of factors are investigated. This is the most efficient design when an experiment requires a study of the effects of two or more factors [1; 3; 5]. It is an experiment whose design consist of two or more factors, each with discrete possible values or levels, and whose experimental units take in all possible combinations of these levels across all such factors [2]. It allows studying the effect of each factor on the response variable, as well as the effects of interactions between factors on the response variable [2; 6].

Factorial experimentation is a method in which the effect due to each factor and combination of factors are estimated [3; 8]. The effect of a factor is the change in response produced by a change in the level of the factor; this is referred to as the main effect [8]. In some experiments, the difference in the response between levels of one factor is not the same at all levels of the other factors; this is called interaction effect [7; 9]. Collectively, main effects and interaction effects are called the factorial effects [3]. In a factorial design, since multiple factors are investigated simultaneously, factors that have significant effects on the response are restricted, as well as interactions [10; 11; 14; 15]. Predictions are also performed (where quantitative factors are present), but care must be taken as certain designs are very limited in the choice of the prediction model [11].

Factorial designs have some advantages over the one-factor design, since it allows effects of a factor to be estimated at several levels of other factors, yielding conclusions that are valid over a range of experimental conditions [10; 11]. Secondly, they are much more efficient for estimating main effects, which are the average effects of a single factor over all units [14; 15].

Design of experiments (factorial design) was first developed by Ronald A. Fisher at the Rothamsted Agricultural Field Research Station in London [12; 13]. His initial experiments were concerned with determining the effects of various fertilizers on different plots of land [13]. In another of his experiments, he showed that they are advantages by combining the study of multiple variables in the same factorial experiment [12; 13]. Since then, deign of factorial experiments has been widely accepted and applied in agriculture, education, biology, statistics, pharmaceutical industry, manufacturing, risk management, chemical process design, engineering, and many other areas [14].

It is pertinent to note that factorial design collects data at the vertices of a cube in $k$-dimensions ($k$ being the number of factors studied), and if the data are collected from all of the vertices, the design is a full (or complete) factorial requiring $2^k$ or $3^k$ runs [2]. This implies that factorial experiments can be performed at two levels, three levels, up to $n$ levels. Moreover, when the total number of all combinations increases exponentially with the number of factors studied (or if the number of combinations in a full factorial design is too high to be logically feasible), a fractional factorial design may be performed, where some of the possible combinations (at least half) are omitted [3; 4]. Two level fractional factorial design is a special category of two-level designs where not all factor level combinations are considered, and the experimenter can choose which combinations are to be excluded. Based on the excluded combinations, certain interactions cannot be determined [3; 5; 6]. However, the price we pay for utilizing a half fractional factorial design is that the main effect of the last factor is aliased with the interaction because they are identical in the model. Additionally, there is also aliasing among other effects, and effects aliasing is a consequence of using a fractional factorial design. A related concept is resolution, which captures the amount of aliasing or confounding [5; 6].

In general, the higher the resolution of a fractional factorial design, the less restrictive the assumption that higher order interactions are negligible to obtain a unique interpretation of a data [13; 14]. Other categories of fractional factorial design include: Plackett-Burman design and Taguchi Orthogonal Arrays [4]. Factorial designs, including fractional factorials, have increased precision over other types of designs because they have built-in internal replication [2; 5; 6]. Replicates of the same points are not needed in a factorial design, which seems like violation of the replication principle; replication is also provided by the factors included in the design turns out to have non-significant effects [7]. In this paper, we have applied fractional factorial design to studying and improving crop yield using different fertilizer combinations.

### *1.2 Statement of the problem*

Over the years, experimenters have studied three-level factorial experiments up to $\boldsymbol{k}$-factors but no work has been done for the design layout and derivation of model parameters for $\mathbf{3^4}$ factorial design. Moreover, developing a suitable defining contrast that will help in constructing the desired number of blocks is another problem often encountered in fractional factorial design, and this is what we sought to address in this paper.

### *1.3 Aim and objectives of the study*

The aim of this paper was to obtain a suitable defining contrast that will help in confounding a three-level fractional factorial experiment, and to as well select an experimental design that allows clear evaluation of main effects and interaction effects (especially two-factor interactions). In line with achieving the stated aim, the objectives of the study were to: (i) define a suitable defining (simultaneous) contrast that will help in obtaining the required number of blocks, (ii) construct a fractional factorial experiment of $3^{4-1}$, (iii) check for aliases of two, three, and four factor interactions, (iv) identify the type of resolution involve in (ii), (v) test for significance of the factor effects in the constructed experiment, and (vi) draw conclusions based on our findings.

## 2. MATERIALS AND METHODS

### *2.1 The $3^k$ factorial design*

This is a factorial arrangement with $k$ factors each at three levels, where factors and interactions are denoted by capital letters, while the three levels are denoted 0, 1 and 2 for low, intermediate and high levels, respectively [2; 3]. Each treatment combination in the design will be denoted by $k$ digits, with the first digit indicating the level of factor $A$, the second digit, the level of factor $B$, and the $k^{th}$ digit, the level of factor $k$ [5].

There are $3^k$ treatment combinations or runs with $3^k - 1$ degrees of freedom between them. These treatment combinations allow sum of squares to be determined for $k$ main effects, each with two degrees of freedom, $\binom{k}{2}$ two-factor interactions, each with four degrees of freedom, and so on; and one $k$ -factor interaction with $2^k$ degrees of freedom [3; 5]. For a $3^4$ factorial design with 81 runs, the treatment combinations allow sum of squares to be determined for 4 main effects, each with two degrees of freedom, 6 two-factor interactions, each with four degrees of freedom, 4 three-factor interactions, each with eight degrees of freedom, and one four-factor interaction with $2^4$ degrees of freedom [14].

In general, an $h$ -factor interaction has $2^h$ degrees of freedom [2; 5]. If there are $n$ replicates, there are $n3^k - 1$ total degrees of freedom and $3^k(n-1)$ degrees of freedom for the error. Sum of squares for effects and interactions in the $3^k$ factorial designs are computed by the usual methods for $2^k$ factorial designs [2].

Aside the fact that each three-level factor has two degrees of freedom, there are two systems for parameterizing the interaction effects: the orthogonal components system and the linear-quadratic system. Standard analysis of variance is applicable to the orthogonal components systems, while a new regression analysis strategy is developed for the linear-quadratic systems [12, 13].

Alternatively, if we denote the $3^k$ treatment combinations with $k$ factors by $A_1, A_2, \dots, A_n$ where factor $A_i$ has $a_{i0}$, $a_{i1}$, and $a_{i2}(i = 1,2, \dots, n)$ or simply $x_i = 0, 1, 2$ [5; 6; 9]. Then, a treatment combination can be expressed in the condensed form as:

$$a_1^{x_1} a_2^{x_2} \dots a_k^{x_k} \tag{1}$$

where $x_i = 0,1,2$ $(i = 1,2, \dots, k)$ and the $x$ representation form as:

$$(x_1, x_2, \dots, x_k),\ x_i = 0, 1, 2 \tag{2}$$

The general expression for a partition of the $3^k$ treatment combinations into three sets of $3^{k-1}$ treatment combinations each is given by:

$$\alpha_1 x_1 + \alpha_2 x_2 + \cdots + \alpha_k x_k = 0, 1, 2\ mod\ 3 \tag{3}$$

with $\alpha_i = 0,1$ and $x_i = 0,1,2(i = 1,2, \dots, k)$. We write the effects and interactions with $\frac{(3^k-1)}{2}$ symbols in general form as:

$$A_1^{\alpha_1} A_2^{\alpha_2} \dots A_k^{\alpha_k} \tag{4}$$

Where any letter $A_i$ with $\alpha_i = 0$ is dropped from the expression, and the first nonzero $\alpha$ equals one can be achieved by multiplying each $\alpha_i$ by 2. We also note that any $\alpha_i = 1$ is not written explicitly in the expression.

### *2.2 Parameterization in terms of main effects and interactions*

If $E^{\alpha} = A_1^{\alpha_1} A_2^{\alpha_2} \dots A_k^{\alpha_k}$ represents an interaction, then

$$E_i^{\alpha} = \left(A_1^{\alpha_1} A_2^{\alpha_2} \dots A_k^{\alpha_k}\right)_i = \begin{pmatrix} mean\ of\ treatment\ combinations \\ satisfying\ \alpha' x = i\ mod\ 3 \end{pmatrix} - M \quad (5)$$

We shall also use the notation $E_{\alpha' x}^{\alpha}$ for given $\alpha$ and $x$ to denote of the quantities $E_0^{\alpha}$, $E_1^{\alpha}$, and $E_2^{\alpha}$ depending on whether $\alpha' x = 0,1,2\ mod\ 3$, respectively. We note that a comparison belonging to $E^{\alpha}$ is given by:

$$c_0 E_0^{\alpha} + c_1 E_1^{\alpha} + c_2 E_2^{\alpha}\ (c_0 + c_1 + c_2 = 0) \quad (6)$$

Also, it follows that:

$$E_0^{\alpha} + E_1^{\alpha} + E_2^{\alpha} = 0 \quad (7)$$

So that any comparison of the form above could be expressed in terms of only two $E_i^{\alpha}$**.** The response $a(x)$ of a treatment combination $x$ as a linear combination of interaction components has the parameterization

$$a(x) = M + \sum_{\alpha} E_{\alpha' x}^{\alpha} \quad (8)$$

where summation is over all $\alpha' = (\alpha_1, \alpha_2, \dots, \alpha_k) \neq (0,0,\dots,0)$, subject to the rule that the first nonzero $\alpha'$ equals 1, and $\alpha' x$ is reduced mod 3. $x' = (x_1, x_2, \dots, x_k)$ and $a(x) = a_1^{x_1} a_2^{x_2}, \dots, a_k^{x_k}, E^{\alpha} = A_1^{x_1} A_2^{x_2}, \dots, A_k^{x_k}$

$E_i^{\alpha} = \left(A_1^{x_1} A_2^{x_2}, \dots, A_k^{x_k}\right)_i\ i = (0,1,2)$

For a $3^4$ factorial with factors $A, B, C, D$ denoting the true response of the treatment combination $(i, j, k, l)$ by $a_i b_j c_k d_l$, then we can write

$$\begin{aligned} a_i b_j c_k d_l = M &+ A_i + B_j + AB_{i+j} + AB_{i+2j}^2 + C_k + AC_{i+k} + AC_{i+2k}^2 + BC_{j+k} + BC_{j+2k}^2 + ABC_{i+j+k} + ABC_{i+j+2k}^2 \\ &+ AB^2C_{i+2j+k} + AB^2C_{i+2j+2k}^2 + D_i + AD_{i+l} + AD_{i+2l}^2 + BD_{j+l} + BD_{j+2l}^2 + CD_{k+l} + CD_{k+2l}^2 \\ &+ ABD_{i+j+l} + ABD_{i+j+2l}^2 + AB^2D_{i+2j+l} + AB^2D_{i+2j+2l}^2 + ACD_{i+j+l} + ACD_{i+k+2l}^2 + AC^2D_{i+2k+l} \\ &+ AC^2D_{i+2k+2l}^2 + BCD_{j+k+l} + BCD_{j+k+2l}^2 + BC^2D_{j+2k+l} + BC^2D_{j+2k+2l}^2 + ABCD_{i+j+k+l} \\ &+ ABCD_{i+j+k+2l}^2 + ABC^2D_{i+j+2k+l} + AB^2CD_{i+2j+k+l} + AB^2C^2D_{i+2j+2k+l} + AB^2CD_{i+2j+k+2l}^2 \\ &+ ABC^2D_{i+j+2k+2l}^2 + AB^2C^2D_{i+2j+2k+2l}^2 \end{aligned}$$

Hence, the $3^4$ factorial design has 81 treatment combinations each with $\frac{(3^4-1)}{2} = \frac{80}{2} = 40$ symbols. The definition of parameterization in terms of the partitions holds for quantitative and qualitative factors, and same has been used in this study.

### *2.3 Design layout for a $3^4$ factorial design*

From the layout in Table 1, we have used digit notation for the treatment combinations, where 0, 1, 2 represent low level, intermediate level and high level respectively. For instance, 0112 represents treatment combination with factor 'A' at low level, factors 'B' and 'C' at intermediate level, and factor 'D' at high level.

### *2.4 Confounding in the $3^k$ factorial design*

In the $3^k$ factorial design each main effect has two degrees of freedom, and every two-factor interaction has four degrees of freedom, and can be decomposed into two components of interaction, each with two degrees of freedom. Every three-factor interaction has eight degrees of freedom and can be decomposed into four components of interaction, each with two degrees of freedom.

Hence, it becomes necessary or convenient to confound a component of interaction with blocks, and the general procedure is to construct a defining contrast

$$L = \alpha_1 x_1 + \alpha_2 x_2 + \cdots + \alpha_k x_k \quad (9)$$

$\alpha_i$ = the exponent appearing on the $i^{th}$ factor in the effect to be confounded.

$x_i$ = the level of the $i^{th}$ factor appearing in a particular treatment combination

For this design, we have $\alpha_i = 0, 1, 2$ with the first nonzero $\alpha_i$ as unity, and $x_i = 0, 1, 2$ indicating low, intermediate or high levels respectively. The treatment combinations are assigned to three blocks depending on whether it satisfies $L = 0(mod\ 3)$, $L = 1(mod\ 3)$ or $L = 2(mod)3$. The treatment combination satisfying $L = 0(mod\ 3)$ constitute the principal block.

Other methods for identification of confounded effects in factorial experiments include: table of sign method, geometric method, odd or even method, and multiplication method.

TABLE 1

| | | | A | | |
|---|---|---|---|---|---|
| D | C | B | 0 | 1 | 2 |
| | | 0 | 0000 | 1000 | 2000 |
| | 0 | 1 | 0010 | 1010 | 2010 |
| | | 2 | 0020 | 1020 | 2020 |
| | | 0 | 0100 | 1100 | 2100 |
| 0 | 1 | 1 | 0110 | 1110 | 2110 |
| | | 2 | 0120 | 1120 | 2120 |
| | | 0 | 0200 | 1200 | 2200 |
| | 2 | 1 | 0210 | 1210 | 2210 |
| | | 2 | 0200 | 1220 | 2220 |
| | | 0 | 0001 | 1001 | 2001 |
| | 0 | 1 | 0011 | 1011 | 2011 |
| | | 2 | 0021 | 1021 | 2021 |
| | | 0 | 0101 | 1101 | 2101 |
| 1 | 1 | 1 | 0111 | 1111 | 2111 |
| | | 2 | 0121 | 1121 | 2121 |

| | | | A | | |
|---|---|---|---|---|---|
| D | C | B | 0 | 1 | 2 |
| | | 0 | 0201 | 1201 | 2201 |
| | 2 | 1 | 0211 | 1211 | 2211 |
| | | 2 | 0221 | 1221 | 2221 |
| | | 0 | 0002 | 1002 | 2002 |
| | 0 | 1 | 0012 | 1012 | 2012 |
| | | 2 | 0022 | 1022 | 2022 |
| | | 0 | 0102 | 1102 | 2102 |
| 2 | 1 | 1 | 0112 | 1112 | 2112 |
| | | 2 | 0122 | 1122 | 2122 |
| | | 0 | 0202 | 1202 | 2202 |
| | 2 | 1 | 0212 | 1212 | 2212 |
| | | 2 | 0222 | 1222 | 2222 |

### *2.5 Fractional replication of the $3^k$ factorial design*

When the number of factors or number of levels of the factors increases, the number of treatment combinations increases rapidly. Hence, it is not possible to accommodate all these treatment combinations in a single homogeneous block. For this to be possible, we fractionalize the design and confound a component of interaction with blocks.

To construct a $3^{k-1}$ fractional factorial design, we select a two-degree of freedom component of interaction (generally the highest order interaction), and partition the full $3^k$ design into three blocks. Each of these three blocks is a $3^{k-1}$ fractional factorial design, and any one of the blocks may be selected for use.

If $AB^{\alpha_2}C^{\alpha_2} \dots K^{\alpha_k}$ is the component of interaction used to define the blocks, then $I = AB^{\alpha_2}C^{\alpha_2} \dots K^{\alpha_k}$ is the defining relation of the fractional factorial design. We may introduce the $k^{th}$ factor by equating its levels $x_k$ to the appropriate component of the highest order interaction say $AB^{\alpha_2}C^{\alpha_2} \dots (k-1)^{\alpha_{k-1}}$ through the relation

$$x_k = B_1 x_1 + B_2 x_2 + \cdots + B_{k-1} x_{k-1} \quad (10)$$

where: $\beta_i = (3 - \alpha_k)\alpha_i (mod\ 3)\ for\ 1 \leq i \leq k-1$

This yields a design of the highest possible resolution. Each main effects or component of interaction estimated from the $3^{k-1}$ design has two aliases, which may be found by multiplying the effects by both $I$ and $I^2 mod\ 3$.

Every four-factor interaction has sixteen degrees of freedom, and can be decomposed into eight components $ABCD$, $AB^2CD$, $ABC^2D$, $ABCD^2$, $AB^2CD^2$, $AB^2C^2D$, $ABC^2D^2$ and $AB^2C^2D^2$, each with two degrees of freedom. However, any of these components can be used as defining contrast since they are negligible. In this article, the component of interaction used to define the blocks is $I = ABCD^2$.

***2.6 Aliasing pattern and resolution of the experiment***

Knowing the defining contrast allows one to ascertain the resolution of the design and the general confounding (aliasing) pattern. As stated earlier, the defining contrast for any design is determined from the highest order interaction, hence any of these eight four-factor interactions: $ABCD$, $AB^2CD$, $ABC^2D$, $ABCD^2$, $AB^2CD^2$, $AB^2C^2D$, $ABC^2D^2$ and $AB^2C^2D^2$ may be used as defining contrast. For our design (Wu and Hamada, 2000), we have used $I = ABCD^2$ as our defining contrast. Multiplying both sides of the defining contrast by each of the factors $A$, $B$, $C$ and $D$, respectively, we have:

$$I = ABCD^2$$

$$A = A \times ABCD^2 = (A^2BCD^2)^2 = A^4B^2C^2D^4 = AB^2C^2D$$

$$A = A \times (ABCD^2)^2 = A^3B^2C^2D^4 = (B^2C^2D)^2 = BCD^2$$

$$B = B \times ABCD^2 = AB^2CD^2$$

$$B = B \times (ABCD^2)^2 = (A^2B^3C^2D^4)^2 = ACD^2$$

$$C = C \times ABCD^2 = ABC^2D^2$$

$$C = C \times (ABCD^2)^2 = (A^2B^2C^3D^4)^2 = ABD^2$$

$$D = D \times ABCD^2 = ABCD^3 = ABC$$

$$D = D \times (ABCD^2)^2 = (A^2B^2C^2D^2)^2 = ABCD$$

$$AB = AB \times ABCD^2 = (A^2B^2CD^2)^2 = ABC^2D$$

$$AB = AB \times (ABCD^2)^2 = A^3B^3C^2D^4 = (C^2D)^2 = CD^2$$

$$AB^2 = AB^2 \times ABCD^2 = (A^2B^3CD^2)^2 = A^4C^2D^4 = AC^2D$$

$$AB^2 = AB^2 \times (ABCD^2)^2 = A^3B^4C^2D^4 = BC^2D$$

$$AC = AC \times ABCD^2 = (A^2BC^2D^2)^2 = A^4B^2C^4D^4 = AB^2CD$$

$$AC = AC \times (ABCD^2)^2 = A^3B^2C^3D^4 = (B^2D)^2 = BD^2$$

$$AC^2 = AC^2 \times ABCD^2 = (A^2BC^3D^2)^2 = AB^2D$$

$$AC^2 = AC^2 \times (ABCD^2)^2 = (A^3B^2C^4D^4)^2 = BC^2D^2$$

$$AD = AD \times ABCD^2 = (A^2BCD^3)^2 = A^4B^2C^2 = AB^2C^2$$

$$AD = AD \times (ABCD^2)^2 = (A^3B^2C^2D^5)^2 = (B^2C^2D^2)^2 = BCD$$

$$AD^2 = AD^2 \times ABCD^2 = (A^2BCD^4)^2 = AB^2C^2D^2$$

$$AD^2 = AD^2 \times (ABCD^2)^2 = (A^3B^2C^2D^6)^2 = BC$$

$$BC = BC \times ABCD^2 = AB^2C^2D^2$$

$$BC = BC \times (ABCD^2)^2 = (A^2B^3C^3D^4)^2 = AD^2$$

$$BC^2 = BC^2 \times ABCD^2 = AB^2C^3D^2 = AB^2D^2$$

$$BC^2 = BC^2 \times (ABCD^2)^2 = (A^2B^3C^4D^4)^2 = (A^2CD)^2 = AC^2D^2$$

$$BD = BD \times ABCD^2 = AB^2CD^3 = AB^2C$$

$$BD = BD \times (ABCD^2)^2 = A^2B^3C^2D^5 = (A^2C^2D^2)^2 = ACD$$

$$CD = CD \times ABCD^2 = ABC^2D^3 = ABC^2$$

$$CD = CD \times (ABCD^2)^2 = A^2B^2C^3D^5 = (A^2B^2D^2)^2 = ABD$$

From the above, the following effects are aliased:

$$A = BCD^2 = AB^2C^2D$$

$$B = ACD^2 = AB^2CD^2$$

$$C = ABD^2 = ABC^2D^2$$

$$D = ABC = ABCD$$

$$AB = CD^2 = ABC^2D$$

$$AB^2 = AC^2D = BC^2D$$

$$AC = BD^2 = AB^2CD$$

$$AC^2 = AB^2D = BC^2D^2$$

$$AD = AB^2C^2 = BCD$$

$$AD^2 = BC = AB^2C^2D^2$$

$$BC^2 = AB^2D^2 = AC^2D^2$$

$$BD = AB^2C = ACD$$

$$CD = ABC^2 = ABD$$

It is pertinent to note that main effects are aliased with three-factor interactions, while two-factor interactions are aliased with each other. This is a clear definition of resolution $IV$ design written $3_{IV}^{4-1}$.

However, as a consequence of the reduction in the number of treatments included in the experiment, we shall not be able to estimate the effects involving four factor interactions using the fractional set. All the main effects and two-factor interactions can be estimated under assumption that all three-factor and higher-order interactions are negligible. If that be the case then $A$, $B$, $C$, $D$, $AB^2$, $AC^2$, $AD$, $BC^2$, $BD$ and $CD$ can be estimated because they are not aliased with any other main effect or two-factor interaction component (they are clear).

***2.7 Algorithm for constructing fractional factorial design***

Step 1: Specify the values of $p$ and $k$.

Step 2: Write a complete factorial design for $p - k$ factors (containing 0, 1, 2).

Step 3: Add $k$ further columns to be filled in the next steps.

Step 4: Take the defining relation $X$ containing only one letter (say $W$) other than in the already filled columns. If no such relation can be found, multiply existing relations so that the result contains only one new letter.

Step 5: Multiply this relation $I = X$ by the new letter $W$ and use $XW$ in the next step.

Step 6: Calculate the entries of the next column by multiplying the entries of the original columns belonging to $XW$. Add the generated column to the set of filled-in columns.

Step 7: Continue with step (4) as long as all new columns are filled.

***2.8 Model for the $3^4$ factorial design***

The model for a four-factor factorial experiment at three levels is given as:

$$y_{ijklm} = \mu + \alpha_i + \beta_j + \tau_k + \theta_l + (\alpha\beta)_{ij} + (\alpha\tau)_{ik} + (\alpha\theta)_{il} + (\beta\tau)_{jk} + (\beta\theta)_{jl} + (\tau\theta)_{kl}$$

$$+ (\alpha\beta\tau)_{ijk} + (\alpha\beta\theta)_{ijl} + (\alpha\tau\theta)_{ikl} + (\beta\tau\theta)_{jkl} + (\alpha\beta\tau\theta)_{ijkl} + \varepsilon_{ijklm}$$

$$\begin{cases} i = 1, .., a. & factor\ A \\ j = 1, .., b. & factor\ B \\ k = 1, .., c. & factor\ C \\ l = 1, \dots, d. & factor\ D \\ m = 1, \dots, n. & replication \end{cases} \tag{11}$$

where:

$\mu$ = overall (or grand mean)

$\alpha_i$ = effect due to the $i^{th}$ level of factor A

$\beta_i$ = effect due to the $j^{th}$ level of factor B

$\tau_k$ = effect due to the $k^{th}$ level of factor C

$\theta_l$ = effect due to the $l^{th}$ level of factor D

$(\alpha\beta)_{ij}$, $(\alpha\tau)_{ik}$, $(\alpha\theta)_{il}$, $(\beta\tau)_{jk}$, $(\beta\theta)_{jl}$, and $(\tau\theta)_{kl}$ are two-factor effects for factors $AB$, $AC$, $AD$, $BC$, $BD$, and $CD$ respectively.

$(\alpha\beta\tau)_{ijk}$, $(\alpha\beta\theta)_{ijl}$, $(\alpha\tau\theta)_{ikl}$, and $(\beta\tau\theta)_{jkl}$ are the three factor interaction effects for factors $ABC$, $ABD$, $ACD$, and $BCD$, respectively.

$(\alpha\beta\tau\theta)_{ijkl}$ = the four-factor interaction effects for factor $ABCD$.

$\varepsilon_{ijklm}$ = the random error

Constraints:

$\sum_{i=1}^{a} \alpha_i = 0$, $\sum_{j=1}^{b} \beta_j = 0$, $\sum_{k=1}^{c} \tau_k = 0$, $\sum_{l=1}^{d} \theta_l = 0$, $\sum_{i=1}^{a} (\alpha\beta)_{ij} = 0$, $\sum_{j=1}^{b} (\alpha\beta)_{ij} = 0$, $\sum_{i=1}^{a} (\alpha\tau)_{ik} = 0$, $\sum_{k=1}^{c} (\alpha\tau)_{ik} = 0$, $\sum_{j=1}^{b} (\beta\tau)_{jk} = 0$, $\sum_{k=1}^{c} (\beta\tau)_{jk} = 0$, $\sum_{i=1}^{a} (\alpha\beta\tau)_{ijk} = 0$, $\sum_{j=1}^{b} (\alpha\beta\tau)_{ijk} = 0$, $\sum_{k=1}^{c} (\alpha\beta\tau)_{ijk} = 0$, $\sum_{i=1}^{a} \sum_{j=1}^{b} \sum_{k=1}^{c} \sum_{l=1}^{d} (\alpha\beta\tau\theta)_{ijkl} = 0$

### *2.9 Cultural management of Pawpaw (Carica Papaya)*

Pawpaw is a tropical fruit that belongs to the cactus group of plants. Although it needs adequate water supply throughout its life span, it desires loamy soils rich in plant nutrient with good air flow and plenty sunlight. The plant contains a white latex from which papain is extracted. Papain extracted from green mature fruits is used for certain drug production, silk degumming, softening wool, and for beer production.

Papaya is planted or propagated with seeds. Seeds are gotten from selected fruits produced by controlled pollination to ensure the quality and uniformity of the plantings. Three to four seedlings are raised per a plastic bag or container and are planted in an open field when they are 20 cm high.

For open field planting, the land is ploughed and harrowed twice to insure proper irrigation and drainage. Organic fertilizer and manure should be spread and incorporated in the soil during land preparation, while planting distance is $2.5m \times 1.6m$ to $3m \times 2m$.

To minimize flower drop, fruit drop and growth retardation, irrigate the plants before the soil gets dry. Too much watering of plants should be avoided to prevent fungal infection and ensure soil is aerated from time to time through shallow cultivation to avoid root rot.

Fertilization, propping, and weed control are necessary for flower fertilization, fruit development, and crop sanitization.

### *2.10 Method of data collection*

The data type and source of this research were secondary; the data was obtained from the Cross River University of Technology Research Farm, Obubra. Other sources of this research included: textbooks, articles, journals, and materials from the internet.

## 3. RESULTS

### *3.1 Data analysis*

We were interested in carrying out experiment to study the effect of four factors (fertilizers) on the yield of *Carica Papaya* (pawpaw). Before establishing the pawpaw plantation, a representative soil sample was taken and the soil analysis results indicates that the quantities of fertilization needed before planting are Nitrogen(A), Phosphorus(B), Potassium(C) and Manganese(D). Each of these factors was applied into the soil and studied at three levels as shown in Table 2. The experimental plan is a three-level fractional factorial design in 27 runs, each run replicated three times as shown in Table 4, where the three-levels of each factor are represented by 0, 1, 2. The response chosen to be monitored is the number of fruits borne by individual plants.

TABLE 2

| Factor | 0 | 1 | 2 |
|---|---|---|---|
| A (Nitrogen) | 10 | 15 | 20 |
| B (Phosphorus) | 10 | 20 | 20 |

| Factor | 0 | 1 | 2 |
|---|---|---|---|
| C (Potassium) | 5 | 15 | 25 |
| D (Manganese) | 20 | 25 | 30 |

TABLE 3A

| Treatment Combination | | | | | Number of Fruits Per Stand | | | Totals |
|---|---|---|---|---|---|---|---|---|
| Run Order | A | B | C | D | Rep. 1 | Rep. 2 | Rep. 3 | Yield |
| 1 | 0 | 0 | 0 | 1 | 26 | 33 | 29 | 88 |
| 2 | 0 | 0 | 0 | 2 | 14 | 22 | 27 | 63 |
| 3 | 0 | 0 | 0 | 3 | 17 | 24 | 23 | 64 |
| 4 | 0 | 0 | 1 | 1 | 25 | 23 | 22 | 70 |
| 5 | 0 | 0 | 1 | 2 | 27 | 30 | 26 | 83 |
| 6 | 0 | 0 | 1 | 3 | 18 | 20 | 17 | 55 |
| 7 | 0 | 0 | 2 | 1 | 20 | 24 | 20 | 64 |
| 8 | 0 | 0 | 2 | 2 | 21 | 29 | 16 | 66 |
| 9 | 0 | 0 | 2 | 3 | 15 | 19 | 21 | 55 |
| 10 | 0 | 1 | 0 | 1 | 25 | 22 | 24 | 71 |
| 11 | 0 | 1 | 0 | 2 | 28 | 32 | 31 | 91 |
| 12 | 0 | 1 | 0 | 3 | 19 | 20 | 25 | 64 |
| 13 | 0 | 1 | 1 | 1 | 20 | 31 | 22 | 73 |
| 14 | 0 | 1 | 1 | 2 | 23 | 21 | 24 | 68 |
| 15 | 0 | 1 | 1 | 3 | 24 | 22 | 27 | 73 |
| 16 | 0 | 1 | 2 | 1 | 28 | 20 | 22 | 70 |
| 17 | 0 | 1 | 2 | 2 | 22 | 27 | 20 | 69 |
| 18 | 0 | 1 | 2 | 3 | 23 | 20 | 22 | 65 |
| 19 | 0 | 2 | 0 | 1 | 19 | 23 | 24 | 66 |
| 20 | 0 | 2 | 0 | 2 | 20 | 20 | 19 | 59 |

| Treatment Combination | | | | | Number of Fruits Per Stand | | | Totals |
|---|---|---|---|---|---|---|---|---|
| Run Order | A | B | C | D | Rep. 1 | Rep. 2 | Rep. 3 | Yield |
| 21 | 0 | 2 | 0 | 3 | 29 | 31 | 31 | 91 |
| 22 | 0 | 2 | 1 | 1 | 29 | 31 | 29 | 89 |
| 23 | 0 | 2 | 1 | 2 | 22 | 20 | 25 | 67 |
| 24 | 0 | 2 | 1 | 3 | 23 | 27 | 22 | 72 |
| 25 | 0 | 2 | 2 | 1 | 21 | 25 | 24 | 70 |
| 26 | 0 | 2 | 2 | 2 | 28 | 24 | 27 | 79 |
| 27 | 0 | 2 | 2 | 3 | 24 | 25 | 25 | 74 |
| 28 | 1 | 0 | 0 | 1 | 20 | 26 | 22 | 68 |
| 29 | 1 | 0 | 0 | 2 | 34 | 34 | 34 | 102 |
| 30 | 1 | 0 | 0 | 3 | 22 | 31 | 32 | 85 |
| 31 | 1 | 0 | 1 | 1 | 22 | 32 | 34 | 88 |
| 32 | 1 | 0 | 1 | 2 | 17 | 26 | 28 | 71 |
| 33 | 1 | 0 | 1 | 3 | 33 | 32 | 23 | 88 |
| 34 | 1 | 0 | 2 | 1 | 31 | 29 | 29 | 89 |
| 35 | 1 | 0 | 2 | 2 | 16 | 27 | 25 | 68 |
| 36 | 1 | 0 | 2 | 3 | 19 | 20 | 15 | 54 |
| 37 | 1 | 1 | 0 | 1 | 30 | 26 | 22 | 78 |
| 38 | 1 | 1 | 0 | 2 | 25 | 22 | 23 | 70 |
| 39 | 1 | 1 | 0 | 3 | 34 | 34 | 34 | 102 |
| 40 | 1 | 1 | 1 | 1 | 34 | 33 | 33 | 100 |
| 41 | 1 | 1 | 1 | 2 | 19 | 28 | 19 | 66 |
| 42 | 1 | 1 | 1 | 3 | 23 | 34 | 21 | 78 |
| 43 | 1 | 1 | 2 | 1 | 20 | 25 | 22 | 67 |
| 44 | 1 | 1 | 2 | 2 | 31 | 27 | 32 | 90 |

| Treatment Combination | | | | | Number of Fruits Per Stand | | | Totals |
|---|---|---|---|---|---|---|---|---|
| Run Order | A | B | C | D | Rep. 1 | Rep. 2 | Rep. 3 | Yield |
| 45 | 1 | 1 | 2 | 3 | 24 | 25 | 18 | 67 |
| 46 | 1 | 2 | 0 | 1 | 32 | 35 | 33 | 102 |
| 47 | 1 | 2 | 0 | 2 | 25 | 27 | 30 | 82 |
| 48 | 1 | 2 | 0 | 3 | 19 | 21 | 32 | 72 |
| 49 | 1 | 2 | 1 | 1 | 18 | 22 | 23 | 63 |
| 50 | 1 | 2 | 1 | 2 | 34 | 35 | 25 | 94 |
| 51 | 1 | 2 | 1 | 3 | 33 | 20 | 25 | 78 |
| 52 | 1 | 2 | 2 | 1 | 20 | 23 | 30 | 73 |
| 53 | 1 | 2 | 2 | 2 | 25 | 25 | 24 | 74 |
| 54 | 1 | 2 | 2 | 3 | 25 | 28 | 28 | 81 |
| 55 | 2 | 0 | 0 | 1 | 23 | 24 | 18 | 65 |
| 56 | 2 | 0 | 0 | 2 | 18 | 22 | 33 | 73 |
| 57 | 2 | 0 | 0 | 3 | 36 | 34 | 31 | 101 |
| 58 | 2 | 0 | 1 | 1 | 35 | 35 | 35 | 105 |
| 59 | 2 | 0 | 1 | 2 | 29 | 21 | 29 | 79 |
| 60 | 2 | 0 | 1 | 3 | 23 | 20 | 19 | 62 |
| 61 | 2 | 0 | 2 | 1 | 21 | 31 | 23 | 75 |
| 62 | 2 | 0 | 2 | 2 | 34 | 36 | 33 | 103 |
| 63 | 2 | 0 | 2 | 3 | 25 | 33 | 22 | 80 |
| 64 | 2 | 1 | 0 | 1 | 36 | 35 | 35 | 106 |
| 65 | 2 | 1 | 0 | 2 | 21 | 21 | 16 | 58 |
| 66 | 2 | 1 | 0 | 3 | 20 | 30 | 22 | 72 |
| 67 | 2 | 1 | 1 | 1 | 19 | 27 | 20 | 66 |
| 68 | 2 | 1 | 1 | 2 | 35 | 24 | 36 | 95 |

| Treatment Combination | | | | | Number of Fruits Per Stand | | | Totals |
|---|---|---|---|---|---|---|---|---|
| Run Order | A | B | C | D | Rep. 1 | Rep. 2 | Rep. 3 | Yield |
| 69 | 2 | 1 | 1 | 3 | 21 | 27 | 17 | 65 |
| 70 | 2 | 1 | 2 | 1 | 23 | 32 | 25 | 80 |
| 71 | 2 | 1 | 2 | 2 | 24 | 25 | 26 | 75 |
| 72 | 2 | 1 | 2 | 3 | 31 | 31 | 32 | 94 |
| 73 | 2 | 2 | 0 | 1 | 20 | 35 | 22 | 77 |
| 74 | 2 | 2 | 0 | 2 | 36 | 34 | 34 | 104 |
| 75 | 2 | 2 | 0 | 3 | 23 | 21 | 28 | 72 |
| 76 | 2 | 2 | 1 | 1 | 21 | 25 | 23 | 69 |
| 77 | 2 | 2 | 1 | 2 | 25 | 20 | 24 | 69 |
| 78 | 2 | 2 | 1 | 3 | 35 | 36 | 34 | 105 |
| 79 | 2 | 2 | 2 | 1 | 35 | 35 | 35 | 105 |
| 80 | 2 | 2 | 2 | 2 | 22 | 21 | 26 | 69 |
| 81 | 2 | 2 | 2 | 3 | 21 | 18 | 25 | 64 |

Because of run size economy, cost and difficulty in estimating all the parameters of the model, we had to use a three-level fractional factorial design in 27 runs (one-third fraction). For this to be possible, we needed a defining contrast. This defining contrast was determined from the highest order interactions, and the highest order interaction used was $I = ABCD^2$. This is illustrated as shown below.

TABLE 3B

| Factor | | | | | Totals | |
|---|---|---|---|---|---|---|
| Run Order | A | B | C | D | Yield | $I = ABCD^2 = x_1 + x_2 + x_3 + 2x_4$ |
| 1 | 0 | 0 | 0 | 1 | 88 | 1(0)+1(0)+1(0)+2(0)=0 |
| 2 | 0 | 0 | 0 | 2 | 63 | 1(0)+1(0)+1(0)+2(1)=2 |
| 3 | 0 | 0 | 0 | 3 | 64 | 1(0)+1(0)+1(0)+2(2)=1 |
| 4 | 0 | 0 | 1 | 1 | 70 | 1(0)+1(0)+1(1)+2(0)=1 |
| 5 | 0 | 0 | 1 | 2 | 83 | 1(0)+1(0)+1(1)+2(1)=0 |
| 6 | 0 | 0 | 1 | 3 | 55 | 1(0)+1(0)+1(1)+2(2)=2 |

| Factor | | | | | Totals | |
|---|---|---|---|---|---|---|
| Run Order | A | B | C | D | Yield | $I = ABCD^2 = x_1 + x_2 + x_3 + 2x_4$ |
| 7 | 0 | 0 | 2 | 1 | 64 | 1(0)+1(0)+1(2)+2(0)=2 |
| 8 | 0 | 0 | 2 | 2 | 66 | 1(0)+1(0)+1(2)+2(1)=1 |
| 9 | 0 | 0 | 2 | 3 | 55 | 1(0)+1(0)+1(2)+2(2)=0 |
| 10 | 0 | 1 | 0 | 1 | 71 | 1(0)+1(1)+1(0)+2(0)=1 |
| 11 | 0 | 1 | 0 | 2 | 91 | 1(0)+1(1)+1(0)+2(1)=0 |
| 12 | 0 | 1 | 0 | 3 | 64 | 1(0)+1(1)+1(0)+2(2)=2 |
| 13 | 0 | 1 | 1 | 1 | 73 | 1(0)+1(1)+1(1)+2(0)=2 |
| 14 | 0 | 1 | 1 | 2 | 68 | 1(0)+1(1)+1(1)+2(1)=1 |
| 15 | 0 | 1 | 1 | 3 | 73 | 1(0)+1(1)+1(1)+2(2)=0 |
| 16 | 0 | 1 | 2 | 1 | 70 | 1(0)+1(1)+1(2)+2(0)=0 |
| 17 | 0 | 1 | 2 | 2 | 69 | 1(0)+1(1)+1(2)+2(1)=2 |
| 18 | 0 | 1 | 2 | 3 | 65 | 1(0)+1(1)+1(2)+2(2)=1 |
| 19 | 0 | 2 | 0 | 1 | 66 | 1(0)+1(2)+1(0)+2(0)=2 |
| 20 | 0 | 2 | 0 | 2 | 59 | 1(0)+1(2)+1(0)+2(1)=1 |
| 21 | 0 | 2 | 0 | 3 | 91 | 1(0)+1(2)+1(0)+2(2)=0 |
| 22 | 0 | 2 | 1 | 1 | 89 | 1(0)+1(2)+1(1)+2(0)=0 |
| 23 | 0 | 2 | 1 | 2 | 67 | 1(0)+1(2)+1(1)+2(1)=2 |
| 24 | 0 | 2 | 1 | 3 | 72 | 1(0)+1(2)+1(1)+2(2)=1 |
| 25 | 0 | 2 | 2 | 1 | 70 | 1(0)+1(2)+1(2)+2(0)=1 |
| 26 | 0 | 2 | 2 | 2 | 79 | 1(0)+1(2)+1(2)+2(1)=0 |
| 27 | 0 | 2 | 2 | 3 | 74 | 1(0)+1(2)+1(2)+2(2)=2 |
| 28 | 1 | 0 | 0 | 1 | 68 | 1(1)+1(0)+1(0)+2(0)=1 |
| 29 | 1 | 0 | 0 | 2 | 102 | 1(1)+1(0)+1(0)+2(1)=0 |
| 30 | 1 | 0 | 0 | 3 | 85 | 1(1)+1(0)+1(0)+2(2)=2 |

| Factor | | | | | Totals | |
|---|---|---|---|---|---|---|
| Run Order | A | B | C | D | Yield | $I = ABCD^2 = x_1 + x_2 + x_3 + 2x_4$ |
| 31 | 1 | 0 | 1 | 1 | 88 | 1(1)+1(0)+1(1)+2(0)=2 |
| 32 | 1 | 0 | 1 | 2 | 71 | 1(1)+1(0)+1(1)+2(1)=1 |
| 33 | 1 | 0 | 1 | 3 | 88 | 1(1)+1(0)+1(1)+2(2)=1 |
| 34 | 1 | 0 | 2 | 1 | 89 | 1(1)+1(0)+1(2)+2(0)=0 |
| 35 | 1 | 0 | 2 | 2 | 68 | 1(1)+1(0)+1(2)+2(1)=2 |
| 36 | 1 | 0 | 2 | 3 | 54 | 1(1)+1(0)+1(2)+2(2)=1 |
| 37 | 1 | 1 | 0 | 1 | 78 | 1(1)+1(1)+1(0)+2(0)=2 |
| 38 | 1 | 1 | 0 | 2 | 70 | 1(1)+1(1)+1(0)+2(1)=1 |
| 39 | 1 | 1 | 0 | 3 | 102 | 1(1)+1(1)+1(0)+2(2)=0 |
| 40 | 1 | 1 | 1 | 1 | 100 | 1(1)+1(1)+1(1)+2(0)=0 |
| 41 | 1 | 1 | 1 | 2 | 66 | 1(1)+1(1)+1(1)+2(1)=2 |
| 42 | 1 | 1 | 1 | 3 | 78 | 1(1)+1(1)+1(1)+2(2)=1 |
| 43 | 1 | 1 | 2 | 1 | 67 | 1(1)+1(1)+1(2)+2(0)=1 |
| 44 | 1 | 1 | 2 | 2 | 90 | 1(1)+1(1)+1(2)+2(1)=0 |
| 45 | 1 | 1 | 2 | 3 | 67 | 1(1)+1(1)+1(2)+2(2)=2 |
| 46 | 1 | 2 | 0 | 1 | 102 | 1(1)+1(2)+1(0)+2(0)=0 |
| 47 | 1 | 2 | 0 | 2 | 82 | 1(1)+1(2)+1(0)+2(1)=2 |
| 48 | 1 | 2 | 0 | 3 | 72 | 1(1)+1(2)+1(0)+2(2)=1 |
| 49 | 1 | 2 | 1 | 1 | 63 | 1(1)+1(2)+1(1)+2(0)=1 |
| 50 | 1 | 2 | 1 | 2 | 94 | 1(1)+1(2)+1(1)+2(1)=0 |
| 51 | 1 | 2 | 1 | 3 | 78 | 1(1)+1(2)+1(1)+2(2)=2 |
| 52 | 1 | 2 | 2 | 1 | 73 | 1(1)+1(2)+1(2)+2(0)=2 |
| 53 | 1 | 2 | 2 | 2 | 74 | 1(1)+1(2)+1(2)+2(1)=1 |
| 54 | 1 | 2 | 2 | 3 | 81 | 1(1)+1(2)+1(2)+2(2)=0 |

| Factor | | | | | Totals | |
|---|---|---|---|---|---|---|
| Run Order | A | B | C | D | Yield | $I = ABCD^2 = x_1 + x_2 + x_3 + 2x_4$ |
| 55 | 2 | 0 | 0 | 1 | 65 | 1(2)+1(0)+1(0)+2(0)=2 |
| 56 | 2 | 0 | 0 | 2 | 73 | 1(2)+1(0)+1(0)+2(1)=1 |
| 57 | 2 | 0 | 0 | 3 | 101 | 1(2)+1(0)+1(0)+2(2)=0 |
| 58 | 2 | 0 | 1 | 1 | 105 | 1(2)+1(0)+1(1)+2(0)=0 |
| 59 | 2 | 0 | 1 | 2 | 79 | 1(2)+1(0)+1(1)+2(1)=2 |
| 60 | 2 | 0 | 1 | 3 | 62 | 1(2)+1(0)+1(1)+2(2)=1 |
| 61 | 2 | 0 | 2 | 1 | 75 | 1(2)+1(0)+1(2)+2(0)=1 |
| 62 | 2 | 0 | 2 | 2 | 103 | 1(2)+1(0)+1(2)+2(1)=0 |
| 63 | 2 | 0 | 2 | 3 | 80 | 1(2)+1(0)+1(2)+2(2)=2 |
| 64 | 2 | 1 | 0 | 1 | 106 | 1(2)+1(1)+1(0)+2(0)=0 |
| 65 | 2 | 1 | 0 | 2 | 58 | 1(2)+1(1)+1(0)+2(1)=2 |
| 66 | 2 | 1 | 0 | 3 | 72 | 1(2)+1(1)+1(0)+2(2)=1 |
| 67 | 2 | 1 | 1 | 1 | 66 | 1(2)+1(1)+1(1)+2(0)=1 |
| 68 | 2 | 1 | 1 | 2 | 95 | 1(2)+1(1)+1(1)+2(1)=0 |
| 69 | 2 | 1 | 1 | 3 | 65 | 1(2)+1(1)+1(1)+2(2)=2 |
| 70 | 2 | 1 | 2 | 1 | 80 | 1(2)+1(1)+1(2)+2(0)=2 |
| 71 | 2 | 1 | 2 | 2 | 75 | 1(2)+1(1)+1(2)+2(1)=1 |
| 72 | 2 | 1 | 2 | 3 | 94 | 1(2)+1(1)+1(2)+2(2)=0 |
| 73 | 2 | 2 | 0 | 1 | 77 | 1(2)+1(2)+1(0)+2(0)=1 |
| 74 | 2 | 2 | 0 | 2 | 104 | 1(2)+1(2)+1(0)+2(1)=0 |
| 75 | 2 | 2 | 0 | 3 | 72 | 1(2)+1(2)+1(0)+2(2)=2 |
| 76 | 2 | 2 | 1 | 1 | 69 | 1(2)+1(2)+1(1)+2(0)=2 |
| 77 | 2 | 2 | 1 | 2 | 69 | 1(2)+1(2)+1(1)+2(1)=1 |
| 78 | 2 | 2 | 1 | 3 | 105 | 1(2)+1(2)+1(1)+2(2)=0 |

| Factor | | | | | Totals | |
|---|---|---|---|---|---|---|
| Run Order | A | B | C | D | Yield | $I = ABCD^2 = x_1 + x_2 + x_3 + 2x_4$ |
| 79 | 2 | 2 | 2 | 1 | 105 | 1(2)+1(2)+1(2)+2(0)=0 |
| 80 | 2 | 2 | 2 | 2 | 69 | 1(2)+1(2)+1(2)+2(1)=2 |
| 81 | 2 | 2 | 2 | 3 | 64 | 1(2)+1(2)+1(2)+2(2)=1 |

From the above, values corresponding to $I = 0\ mod\ 3$ , $I = 1\ mod\ 3$, $and\ I = 2\ mod\ 3$ were put in blocks I, II and III respectively. Hence, the principal block, $I = 0\ mod\ 3$ is

TABLE 4

| Run Order | A | B | C | D | Rep. 1 | Rep. 2 | Rep. 3 | Yield |
|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 | 26 | 33 | 29 | 88 |
| 5 | 0 | 0 | 1 | 1 | 27 | 30 | 26 | 83 |
| 9 | 0 | 0 | 2 | 2 | 15 | 19 | 21 | 55 |
| 11 | 0 | 1 | 0 | 1 | 28 | 32 | 31 | 91 |
| 15 | 0 | 1 | 1 | 2 | 24 | 22 | 27 | 73 |
| 16 | 0 | 1 | 2 | 0 | 28 | 20 | 22 | 70 |
| 21 | 0 | 2 | 0 | 2 | 29 | 31 | 21 | 91 |
| 22 | 0 | 2 | 1 | 0 | 29 | 31 | 29 | 89 |
| 26 | 0 | 2 | 2 | 1 | 28 | 24 | 27 | 79 |
| 29 | 1 | 0 | 0 | 1 | 34 | 34 | 34 | 102 |
| 31 | 1 | 0 | 1 | 2 | 33 | 32 | 23 | 88 |
| 34 | 1 | 0 | 2 | 0 | 31 | 29 | 29 | 89 |
| 39 | 1 | 1 | 0 | 2 | 34 | 34 | 34 | 102 |
| 40 | 1 | 1 | 1 | 0 | 34 | 33 | 33 | 100 |
| 44 | 1 | 1 | 2 | 1 | 33 | 32 | 32 | 97 |
| 46 | 1 | 2 | 0 | 0 | 32 | 35 | 33 | 100 |
| 50 | 1 | 2 | 1 | 1 | 34 | 35 | 25 | 94 |
| 54 | 1 | 2 | 2 | 2 | 25 | 28 | 28 | 81 |

| Run Order | A | B | C | D | Rep. 1 | Rep. 2 | Rep. 3 | Yield |
|---|---|---|---|---|---|---|---|---|
| 57 | 2 | 0 | 0 | 2 | 36 | 34 | 31 | 101 |
| 58 | 2 | 0 | 1 | 0 | 35 | 35 | 35 | 105 |
| 62 | 2 | 0 | 2 | 1 | 34 | 36 | 33 | 103 |
| 64 | 2 | 1 | 0 | 0 | 36 | 35 | 35 | 106 |
| 68 | 2 | 1 | 1 | 1 | 35 | 24 | 36 | 95 |
| 72 | 2 | 1 | 2 | 2 | 31 | 31 | 32 | 94 |
| 74 | 2 | 2 | 0 | 1 | 36 | 34 | 34 | 104 |
| 78 | 2 | 2 | 1 | 2 | 35 | 36 | 34 | 105 |
| 79 | 2 | 2 | 2 | 0 | 35 | 35 | 35 | 105 |
| Replication | | | | | 837 | 834 | 819 | |
| Total | | | | | | | | 2490 |

### *3.2. Preliminary analysis of the data*

Correction factor $(CF) = \frac{y_{....}^2}{abcn} = \frac{(2490)^2}{(3\times3\times3\times3)} = 76544.44$

$$SS_{Total} = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}\sum_{l=1}^{n} y_{ijkl}^2 - CF = 26^2 + 27^2 + 15^2 + \cdots + 35^2 - 76544.44 = 78266 - 76544.44 = 1721.56$$

$$SS_{Rep.} = \frac{R_1^2 + R_2^2 + R_3^2}{abc} - CF = \frac{837^2 + 834^2 + 819^2}{27} - 76544.44 = 6.8933$$

$$SS_{Trt.} = \frac{\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c} y_{ijk}}{n} - CF = \frac{88^2 + 83^2 + \cdots + 105^2}{3} - 76544.44 = 1351.56$$

$$SS_E = SS_{Tot.} - SS_{Rep.} - SS_{Trt.} = 1721.56 - 6.8933 - 1351.56 = 363.1067$$

TABLE 5 Preliminary Analysis of Variance Table

| Source of Variation | Degree of Freedom | Sum of Squares | Mean Square | Computed F-Ratio | Tabular F-Ratio |
|---|---|---|---|---|---|
| Replication | 2 | 6.8933 | 3.4467 | 0.7119 | 3.15 |
| Treatment | 3 | 1351.56 | 450.52 | 93.0557* | 3.15 |
| Error | 75 | 363.1067 | 4.8414 | | |
| Total | 80 | 1721.56 | | | |

The above data was analyzed with the help of MINITAB 18 and the analysis of variance table is displayed below:

General Factorial Regression: YIELD versus A, B, C, D

The following terms cannot be estimated and were removed:

B*C, B*D, C*D

Factor Information

| Factor | Levels | Values |
|---|---|---|
| A | 3 | 0, 1, 2 |
| B | 3 | 0, 1, 2 |
| C | 3 | 0, 1, 2 |
| D | 3 | 0, 1, 2 |

Analysis of Variance

| Source | DF | Adj SS | Adj MS | F-Value | P-Value |
|---|---|---|---|---|---|
| Model | 20 | 1335.85 | 66.793 | 10.39 | 0.000 |
| Linear | 8 | 1106.07 | 138.259 | 21.51 | 0.000 |
| A | 2 | 762.74 | 381.370 | 59.33 | 0.000 |
| B | 2 | 21.63 | 10.815 | 1.68 | 0.195 |
| C | 2 | 232.52 | 116.259 | 18.09 | 0.000 |
| D | 2 | 89.19 | 44.593 | 6.94 | 0.002 |
| 2-Way Interactions | 12 | 229.78 | 19.148 | 2.98 | 0.003 |
| A*B | 4 | 102.52 | 25.630 | 3.99 | 0.006 |
| A*C | 4 | 95.85 | 23.963 | 3.73 | 0.009 |
| A*D | 4 | 31.41 | 7.852 | 1.22 | 0.311 |
| Error | 60 | 385.70 | 6.428 | | |
| Lack-of-Fit | 6 | 15.70 | 2.617 | 0.38 | 0.887 |

| Source | DF | Adj SS | Adj MS | F-Value | P-Value |
|---|---|---|---|---|---|
| Pure Error | 54 | 370.00 | 6.852 | | |
| Total | 80 | 1721.56 | | | |

Model Summary

| S | R-sq | R-sq(adj) | R-sq(pred) |
|---|---|---|---|
| 2.53543 | 77.60% | 70.13% | 59.17% |

Regression Equation

YIELD = 30.741 - 4.111 A_0 + 0.852 A_1 + 3.259 A_2 - 0.593 B_0 - 0.074 B_1 + 0.667 B_2 + 2.037 C_0 + 0.074 C_1 - 2.111 C_2 + 0.815 D_0 + 0.667 D_1 - 1.481 D_2 - 0.926 A*B_0 0 - 0.556 A*B_0 1 + 1.481 A*B_0 2 + 0.000 A*B_1 0 + 1.704 A*B_1 1 - 1.704 A*B_1 2 + 0.926 A*B_2 0 - 1.148 A*B_2 1 + 0.222 A*B_2 2 + 1.333 A*C_0 0 + 0.519 A*C_0 1 - 1.852 A*C_0 2 + 0.148 A*C_1 0 - 0.333 A*C_1 1 + 0.185 A*C_1 2 - 1.481 A*C_2 0 - 0.185 A*C_2 1 + 1.667 A*C_2 2 - 0.000 A*D_0 0 + 0.815 A*D_0 1 - 0.815 A*D_0 2 - 0.296 A*D_1 0 + 0.296 A*D_1 1 - 0.000 A*D_1 2 + 0.296 A*D_2 0 - 1.111 A*D_2 1 + 0.815 A*D_2 2

Fits and Diagnostics for Unusual Observations

| Obs | YIELD | Fit | Resid | Std Resid | |
|---|---|---|---|---|---|
| 6 | 28.00 | 22.85 | 5.15 | 2.36 | R |
| 50 | 24.00 | 32.22 | -8.22 | -3.77 | R |
| 65 | 23.00 | 29.26 | -6.26 | -2.87 | R |
| 71 | 25.00 | 31.26 | -6.26 | -2.87 | R |

*R Large residual*

The same analysis when performed or run in R gave the following results:

| | Df | Sum Sq | Mean Sq | F value | Pr(>F) |
|---|---|---|---|---|---|
| A | 2 | 762.7 | 381.4 | 55.659 | 7.58e-14 *** |
| B | 2 | 21.6 | 10.8 | 1.578 | 0.21568 |
| C | 2 | 232.5 | 116.3 | 16.968 | 1.92e-06 *** |
| D | 2 | 89.2 | 44.6 | 6.508 | 0.00294 ** |
| A:B | 4 | 102.5 | 25.6 | 3.741 | 0.00930 ** |
| A:C | 4 | 95.9 | 24.0 | 3.497 | 0.01304 * |
| A:D | 4 | 31.4 | 7.9 | 1.146 | 0.34495 |
| B:C | 2 | 2.1 | 1.0 | 0.151 | 0.85991 |
| B:D | 2 | 7.6 | 3.8 | 0.557 | 0.57632 |
| C:D | 2 | 6.0 | 3.0 | 0.438 | 0.64770 |
| Residuals | 54 | 370.0 | 6.9 | | --- |

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

### *3.3 Discussion of findings*

Factorial experiments are experiments in which main effects and interactions are studied simultaneously. Factorial design is one in which every possible combination of treatment levels for different factors are considered. Therefore, as the number of factors increases, the treatment combination and or interaction also increases rapidly, and this may be expensive, difficult or sometimes impossible to estimate all the parameters or all the treatment combinations. To keep experimental costs in line, one possible approach is to use fractional factorial designs where one does not take measurements upon every possible combination of factor-levels, but only on a chosen few without losing much information. These few are selected to ensure that the main effects and lower-order interactions can be estimated and tested at the expense of higher-order interactions. For these reasons, we confounded a $3^{4-1}$ fractional factorial design on *Carica Papaya* (pawpaw) via different fertilizer combinations of phosphorus (A), Nitrogen (B), Potassium (C) and Manganese (D), where the number of treatment combination was reduced from 81 to 27 by running a fraction of the complete factorial experiment.

A major characteristic of a fractional design is its resolution – the degree to which main effects and interactions are independently estimated and interpreted [15]. The resolution of a design is determined from the alias structure, and our alias structure indicated that main effects are confounded with three factor interactions, while two factor interactions are confounded with each other. This is a clear definition of Resolution IV designs; hence, the design used is a $3_{IV}^{4-1}$ fractional factorial design.

From the preliminary analysis, the critical value $F_{0.05,3,75} = 3.15$, compared with the calculated values showed that the treatments are significant, but in order to determine which of these treatments (fertilizers) was more important necessitated a formal analysis of the data using Minitab 18 and R statistical software.

The main effects plot suggested that Nitrogen (A) was the most significant, followed by Potassium (C), Phosphorus (B) and Manganese (D). The interaction plots also showed that there may be some interactions, since the lines were not parallel. From the formal analysis in Minitab, the analysis of variance table showed that Nitrogen (A), Potassium (C), Manganese (D), and the interactions: Nitrogen and Phosphorus (AB), and Nitrogen and Potassium (AC), were significant, while other factors (main and interaction effects) were not significant. Moreover. The Pareto chart showed the significance of each factor on the response (yield) of crop. The red line denoted alpha ($\alpha$) set at 0.05 (equivalent to 95% confidence). If a bar crosses the red line, the corresponding effect was said to be significant. Thus, factors A, C, D and interactions AD and AC were deemed significant. This was produced with the help of Minitab 18 statistical software.

## 4. CONCLUSION

Creating large number of treatment combinations is not only complex and cost intensive, it also has a higher-order interactions which most times fail to be significant, and often prove difficult to interpret because many factors are taken into consideration. Even if they are significant, they explain only small portions of variance. For this reason, we recommended the use of fractional factorial design on crop yield using different fertilizer combinations, in which case responses are driven by a limited number of main effects and lower order interactions. We also recommended the proper application of these fertilizer combinations in their right proportion, in order to enhance good yield and production of food to the teeming population in Nigeria. Moreover, we believe that the data layout, estimation and derivation of model parameters for $3^4$ design will be useful to students, as well as scholars in design and analysis of experiments.

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